

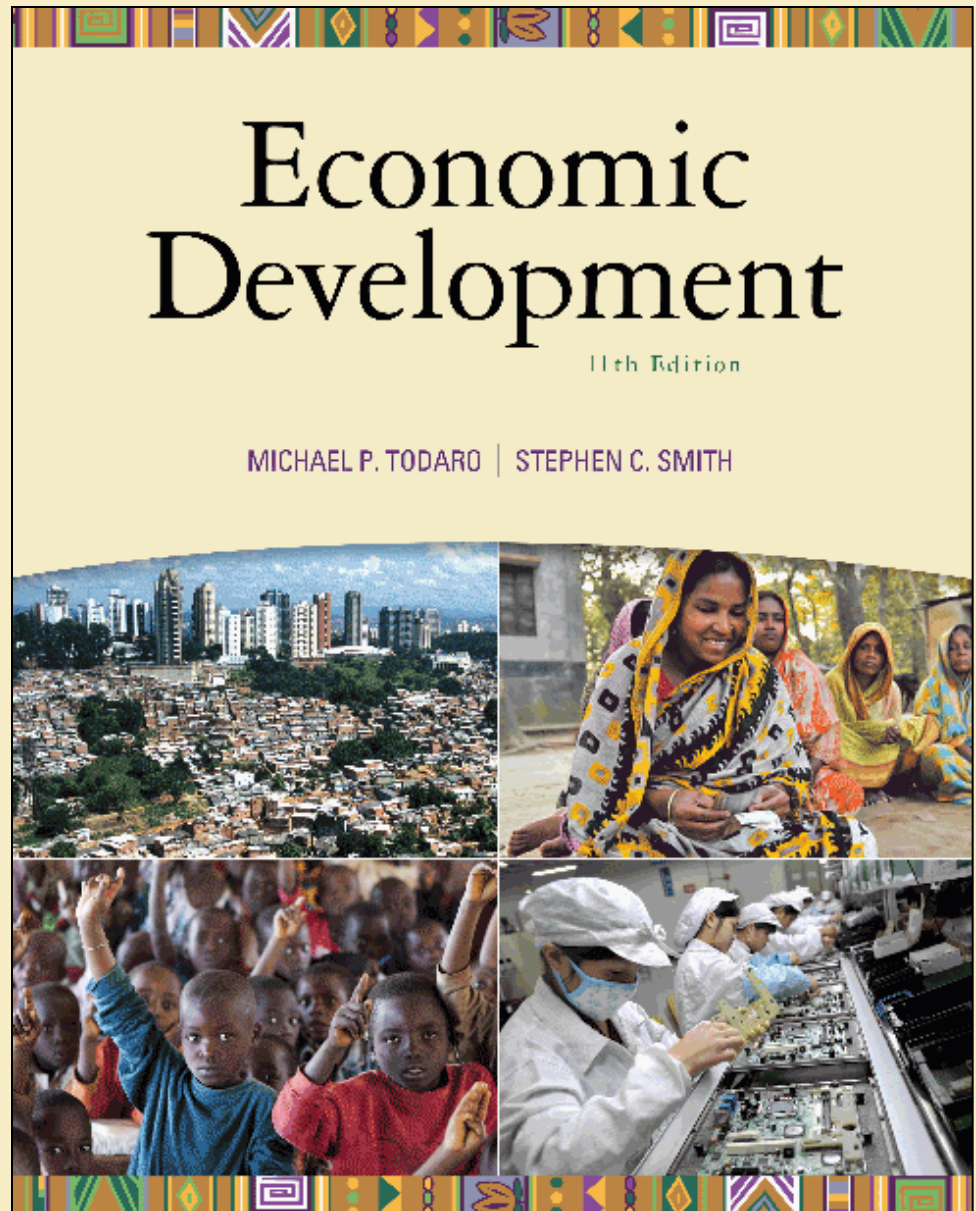
Chapter 3

Classic Theories of Economic Growth and Development

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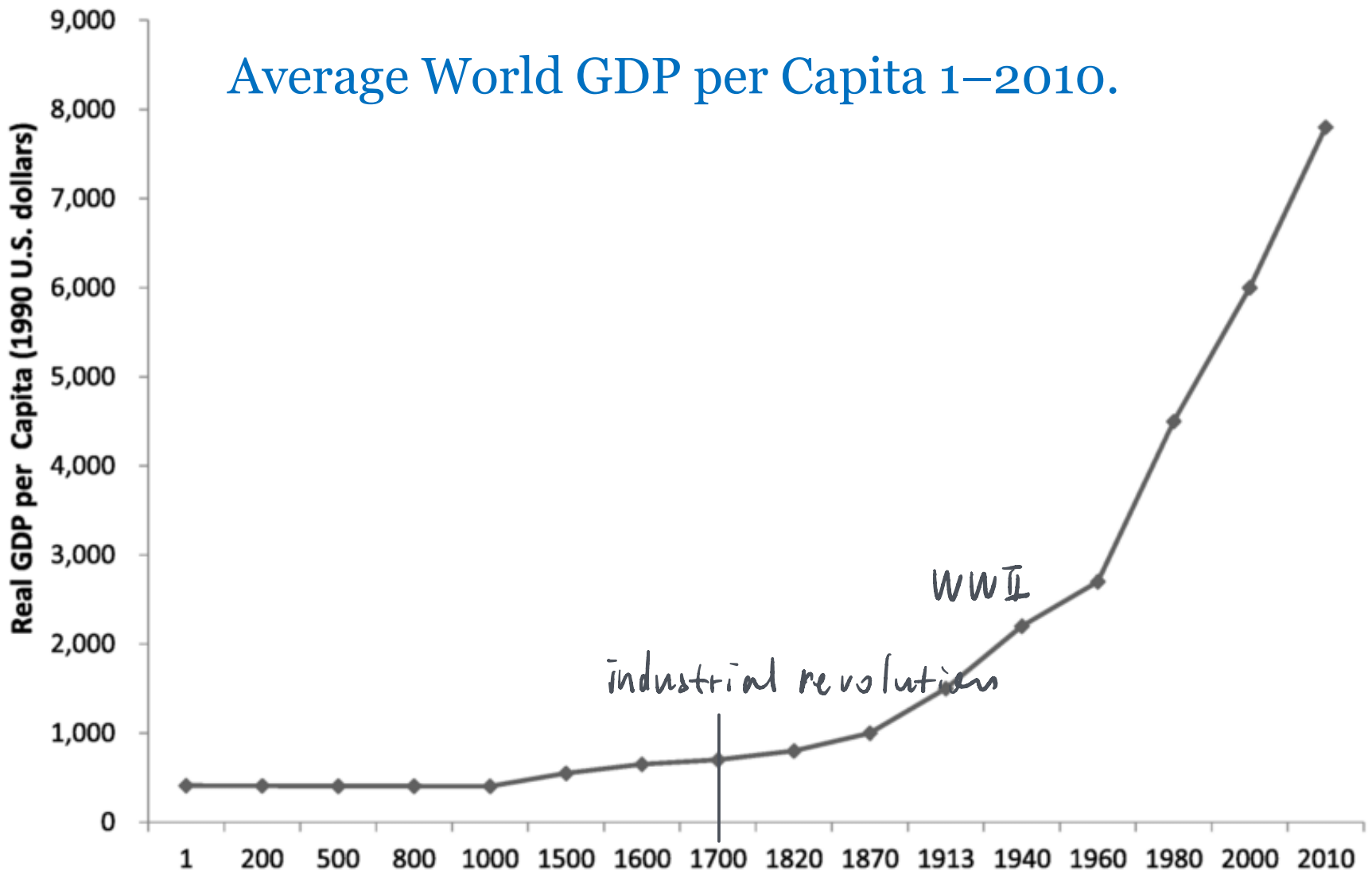
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Growth over the Very Long Run

- Sustained increases in standards of living are a recent phenomenon.
- Sustained economic growth emerges in different places at different times.
 - Thus, per capita GDP differs remarkably around the world.

Average World GDP per Capita 1–2010.



Source: Angus Maddison (2013); Tytell, Emsbo-Mattingly, and Hofschire (2016)

分歧

- The Great Divergence

- **Divergence**: A tendency for per capita income (or output) to grow faster in higher-income countries than in lower-income countries so that the income gap widens across countries over time
- The recent era of increased difference in standards of living across countries.

- Before 1700

- Per capita GDP in nations differed only by a factor of two or three. 2~3倍

- Today

- Per capita GDP differs by a factor of 50 for several countries. 50倍

Modern Growth around the World

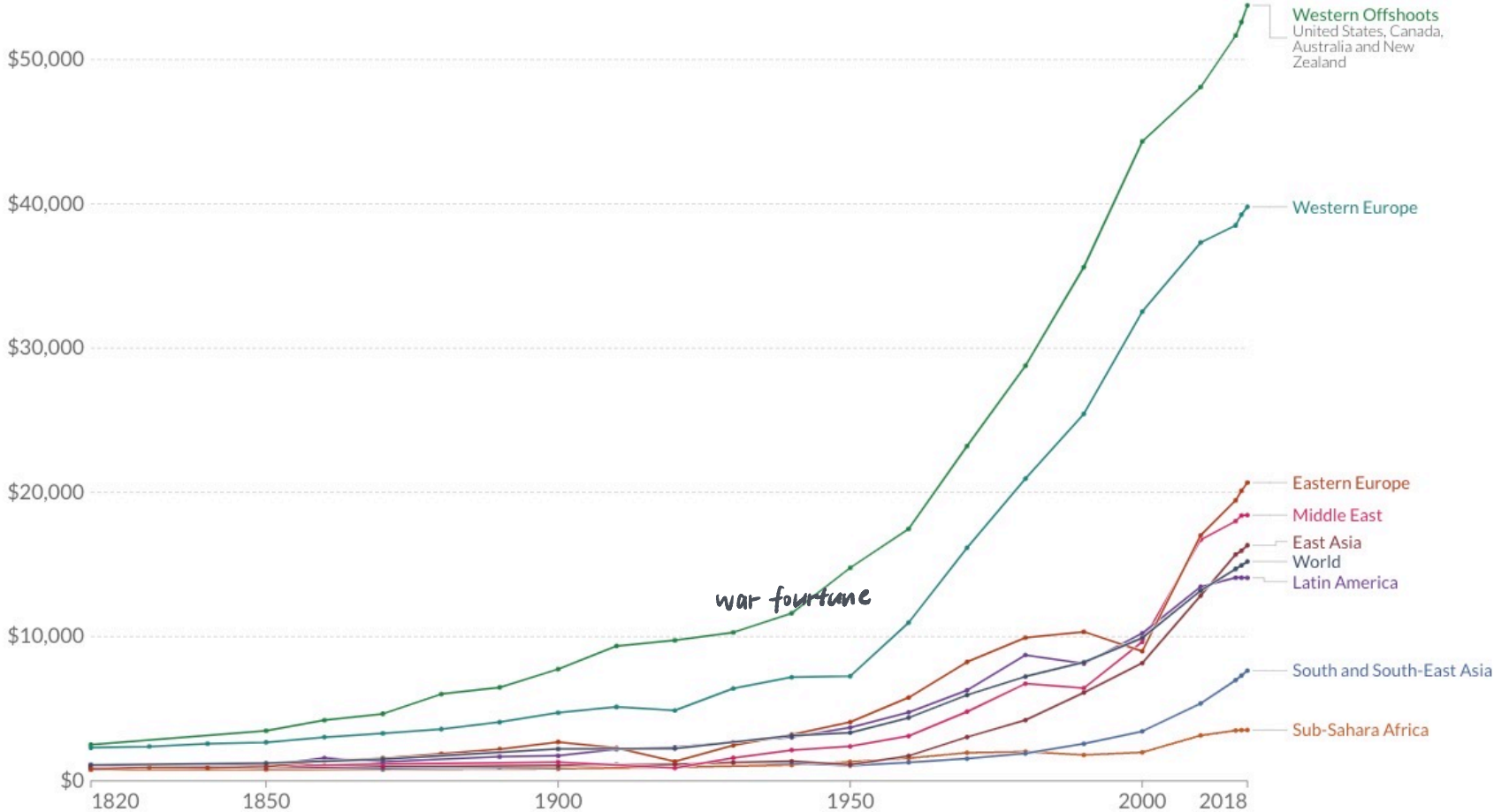
- After World War II, growth in Germany and Japan accelerated.
- **Convergence** 收敛
 - Poorer countries will grow faster to “catch up” to the level of income in richer countries.
- Brazil had accelerated growth until 1980 and then stagnated. 停滞
- China and India have had the reverse pattern.

GDP per capita, 1820 to 2018

This data is adjusted for differences in the cost of living between countries, and for inflation. It is measured in constant 2011 international- $\$$.

Our World
in Data

LINEAR LOG **+ Add country** Relative change



Source: Maddison Project Database 2020 (Bolt and van Zanden, 2020)

OurWorldInData.org/economic-growth • CC BY

▶ 1820 ○ 2018

CHART

MAP

TABLE

SOURCES

DOWNLOAD

🔗

A Broad Sample of Countries

- Over the period 1960–2007
 - Some countries have exhibited a negative growth rate. 负增长
 - Other countries have sustained nearly 6 percent growth.
 - Most countries have sustained about 2 percent growth. e.g. US
- Small differences in growth rates result in large differences in standards of living.

* 70 rules

$$\text{double time : } \frac{70}{6} = 11.7 \text{ years}$$

$$\frac{70}{2} = 35 \text{ years}$$



GDP

GDP growth rate

1,200,000

16.00%

万亿

中国历年国内生产总值 (GDP) 与增长率一览 (2000年—2019年)

增速

1,000,000

14.00%

800,000

12.00%

600,000

10.00%

400,000

8.00%

200,000

6.00%

0

4.00%

2.00%

0.00%

2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 年份

GDP (万亿)

增长率

14.16%

12.68%

11.31%

10.05%

10.09%

9.08%

8.43%

8.30%

9.63%

9.21%

10.45%

9.30%

7.65%

7.67%

7.40%

6.90%

6.70%

6.90%

6.60%

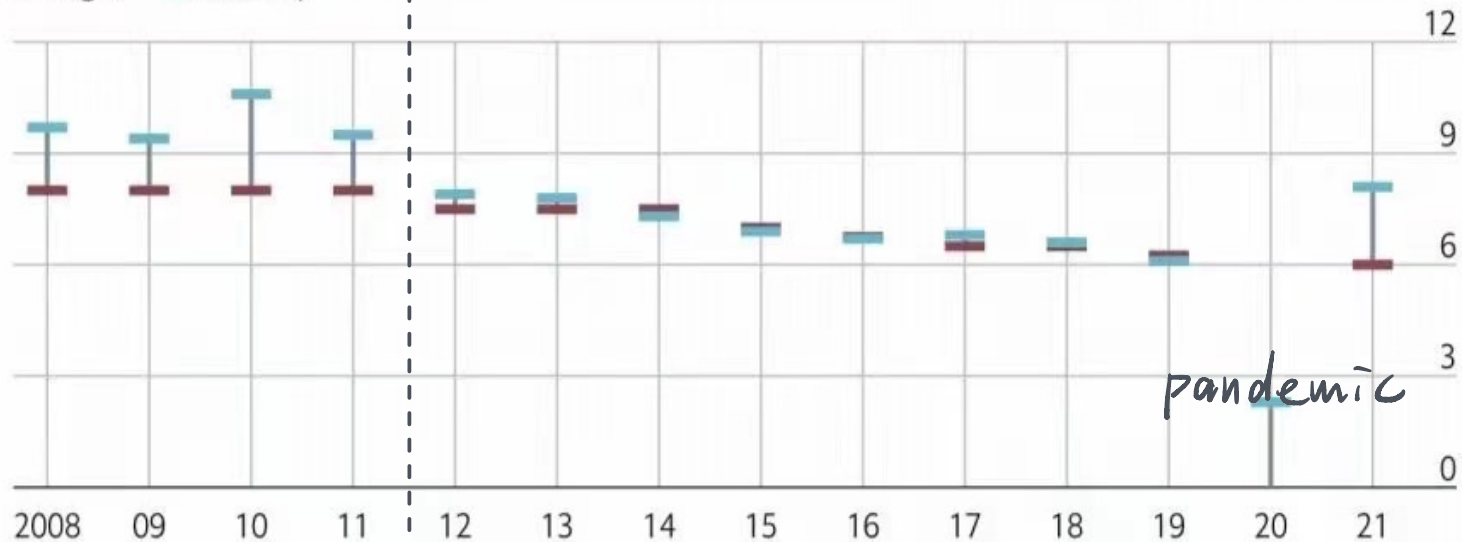
6.10%

financial crisis

Real GDP Growth in China: Target and Actuality

year-on-year in %.

■ Target ■ Actuality



Source: The State Council of China, National Bureau of Statistics of China.

- The growth rate in 2021 : 8.1%, 114, 367 billion yuan
- The target growth rate for 2022 : 5.5%
- The actual growth rate for 2022 : 3% , 121, 021 billion yuan
- The target growth rate for 2023 : 5%
- The actual growth rate for 2023 : 5.2%, 126, 059 billion yuan

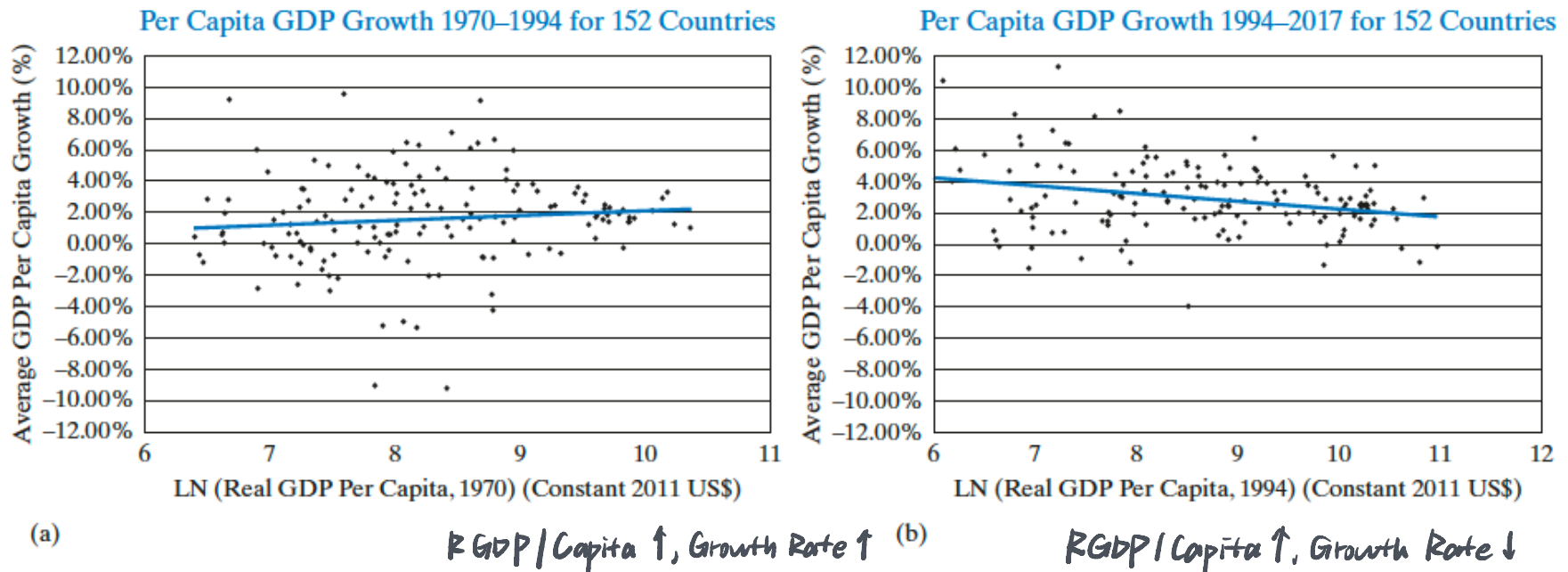
Rank Province GDP23 GDP22 NGR RGR Area Population GDPPC EconIntensity

2023年 排名	省级政区	2023年GDP 总量（初步核 算，亿元）	2022年GDP 总量（最终核 算，亿元）	GDP 名义增速	GDP 实际增速	面积 （平方公里）	2022年底常住 人口（万人）	人均GDP （元/人）	经济密度 （万元/平方公里）
1	广东省	135673	129119	5.1%	4.8%	184800	12656.80	107194	7342
2	江苏省	128222	122089	5.0%	5.8%	102658	8515.00	150584	12490
3	山东省	92069	87577	5.1%	6.0%	157126	10162.79	90594	5860
4	浙江省	82553	78061	5.8%	6.0%	104141	6577.00	125518	7927
5	四川省	60133	56610	6.2%	6.0%	488000	8374.00	71809	1232
6	河南省	59132	58220	1.6%	4.1%	167000	9872.00	59899	3541
7	湖北省	55804	53735	3.8%	6.0%	187400	5844.00	95489	2978
8	福建省	54355	51765	5.0%	4.5%	120000	4188.00	129788	4530
9	湖南省	50013	47559	5.2%	4.6%	211829	6604.00	75731	2361
10	上海市	47219	44809	5.4%	5.0%	6219	2475.89	190714	75926
11	安徽省	47051	45045	4.5%	5.8%	139427	6127.00	76792	3375
12	河北省	43944	42370	3.7%	5.5%	190000	7420.00	59224	2313
13	北京市	43761	41541	5.3%	5.2%	16411	2184.30	200342	26665
14	陕西省	33786	32838	2.9%	4.3%	205800	3956.00	85405	1642
15	江西省	32200	31214	3.2%	4.1%	166900	4527.98	71114	1929
16	辽宁省	30209	28826	4.8%	6.3%	148000	4197.00	71979	2041
17	重庆市	30146	29129	3.5%	6.1%	82400	3213.30	93816	3658
18	云南省	30021	28954	3.7%	4.4%	394000	4693.00	63970	762
19	广西壮族自治区	27202	26186	3.9%	4.1%	236700	5047.00	53898	1149
20	山西省	25698	25584	0.4%	5.0%	156000	3481.35	73817	1647
21	内蒙古自治区	24627	23389	5.3%	7.3%	1183000	2401.17	102563	208
22	贵州省	20913	20165	3.7%	4.9%	170000	3856.00	54236	1230
23	新疆维吾尔自治区	19126	17741	7.8%	6.8%	1664897	2587.00	73931	115
24	天津市	16737	16132	3.8%	4.3%	11920	1363.00	122798	14041
25	黑龙江省	15884	15832	0.3%	2.6%	469000	3099.00	51255	339
26	吉林省	13531	12818	5.6%	6.3%	187000	2347.69	57636	724
27	甘肃省	11864	11121	6.7%	6.4%	454430	2492.00	47608	261
28	海南省	7551	6890	9.6%	9.2%	34000	1027.02	73525	2221
29	宁夏回族自治区	5315	5105	4.1%	6.6%	66400	728.00	73008	800
30	青海省	3799	3623	4.9%	5.3%	721000	593.00	63850	53
31	西藏自治区	2393	2133	12.2%	9.5%	1220000	363.90	65751	20

Reasons to Expect Convergence

- Technology transfer 技术转让
 - Enable developing countries to “leapfrog” over some of the earlier stages of technological development “跳过”.
- Expect convergence if conditions are similar
 - Diminishing returns to factor accumulation ↓
- Divergence occurred for two centuries from the start of the industrial revolution. However, the most recent data demonstrate that, on average, (re-)convergence is now underway. 正在收敛

FIGURE 2.6 Relative Country Convergence 1970–1994 and 1994–2017



Data Source: Penn World Table

In recent years, the pattern of growth across countries has changed so strongly. 增长模式剧变

FIGURE 2.7 Relative Country Convergence: World, Developing Countries, and OECD

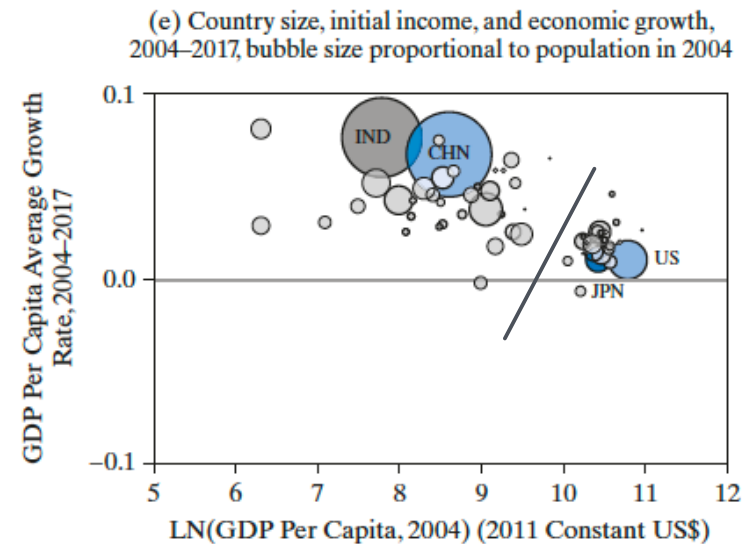
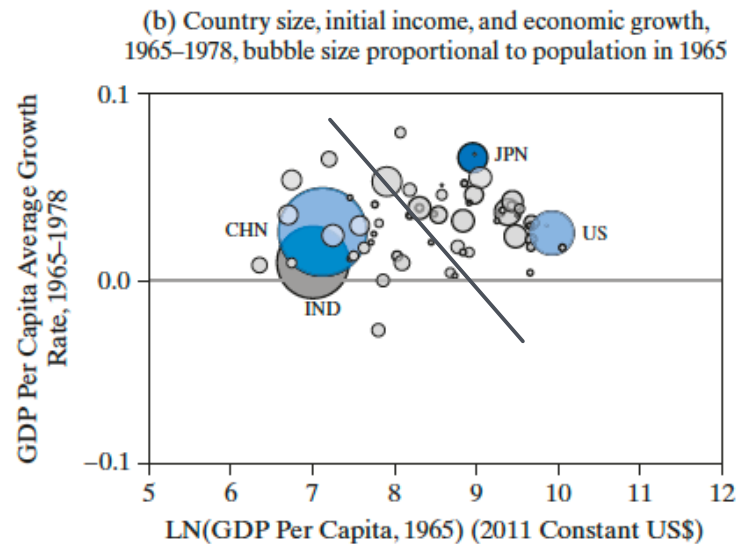
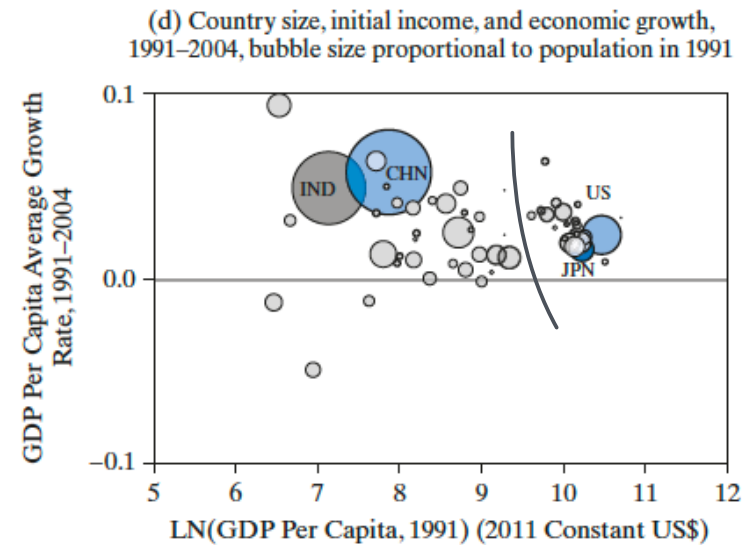
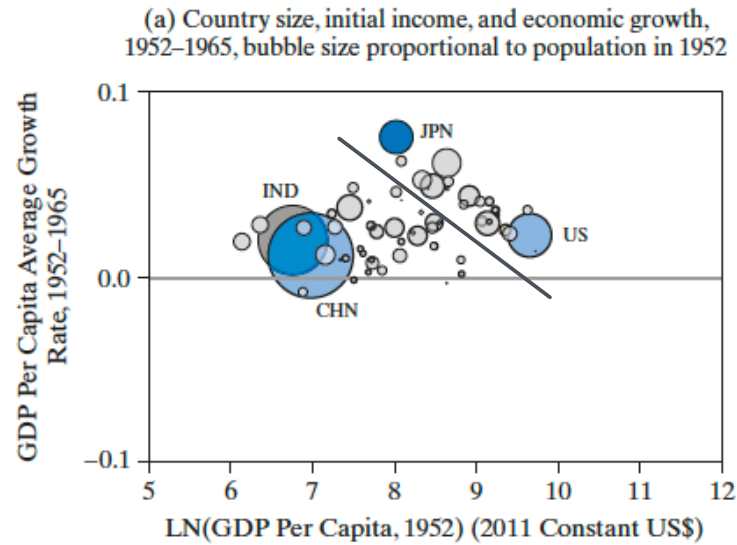
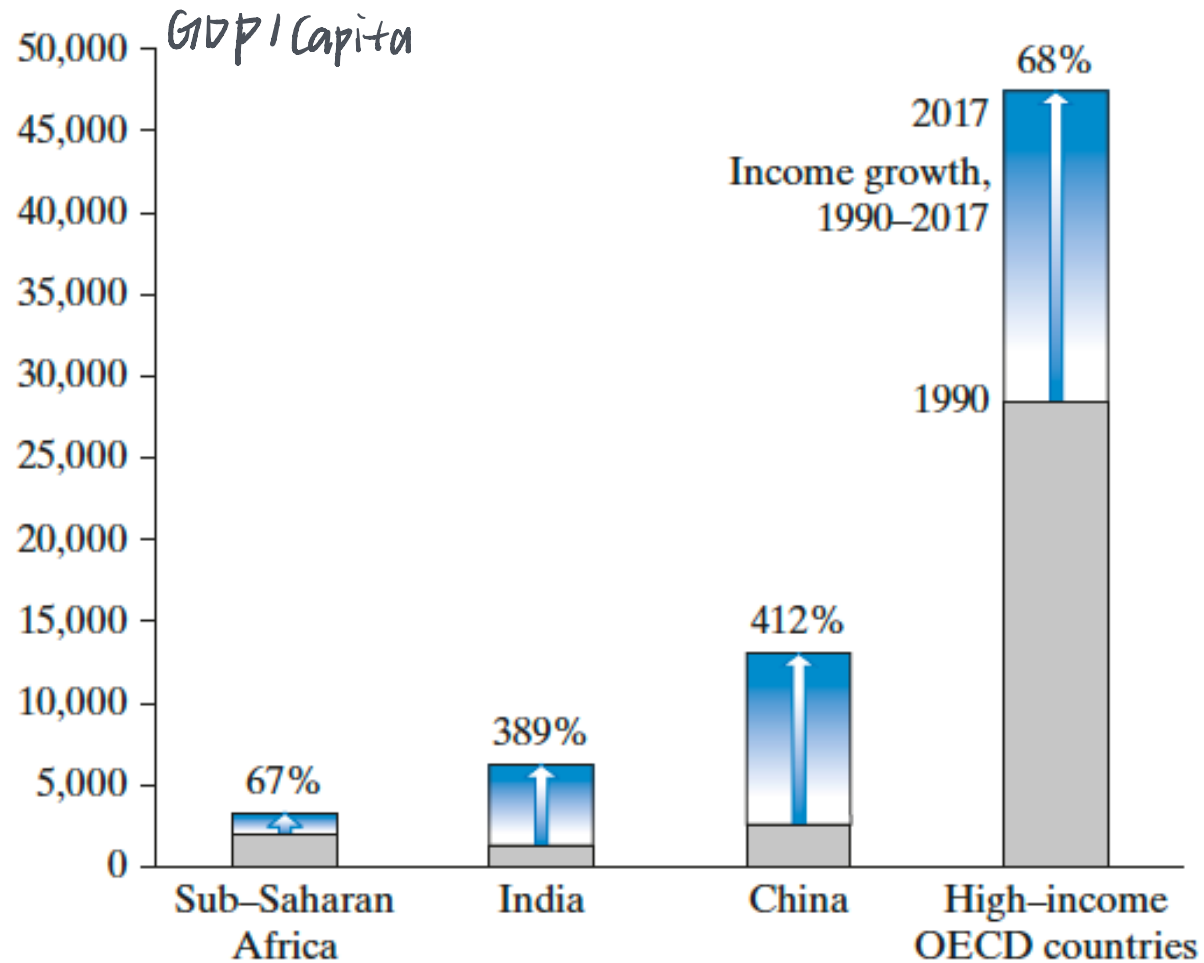


FIGURE 2.8 Growth Convergence versus Absolute Income Convergence



Data Source: Penn World Table

3.1 Classic Theories of Economic Development: Four Approaches

- △ • Linear stages of growth model 阶段增长模型
 - View development as a series of successive stages *AK Model*
- △ • Theories and patterns of structural change 结构变革
 - Use modern economic theory & statistical analysis to portray the internal process of structural change
- International-dependence revolution 国际依附革命理论
 - Emphasize external and internal institutional and political constraints on economic development
- Neoclassical, free market counterrevolution 反革命
 - Emphasize free markets, open economies, and the privatization of inefficient public enterprises *Solow*
私有化

3.2 Development as Growth and Linear- Stages Theories 阶段增长理论

- A Classic Statement: Rostow's Stages of Growth
 - The transition from underdevelopment to development can be described in terms of a series of steps or stages through which all countries must proceed
- Harrod-Domar Growth Model (sometimes referred to as the AK model)
 - New investment to replace worn-out capital

The Harrod-Domar Model

- Background
 - Marshall plan between 1948-51 马歇尔计划
- The model is appealing to scholars and politicians because it is very simple
- Two economists developed the theory independently in 1940s
 - Roy Harrod (England)
 - Evsey Domar (MIT)

Harrod



Domar



The Harrod-Domar Model

- Net saving is some proportion of national income

$$S = sY \quad (3.1) \quad s: \text{saving rate}$$

- Net investment is change in capital stock

$$I = \Delta K \quad (3.2)$$

- Suppose the capital-output ratio is fixed

$$\frac{K}{Y} = c$$

parameter
variable

- Or, $\Delta K = c\Delta Y \quad (3.3)$

- Net national savings must equal net investment

$$S = I \quad (3.4)$$

- Now solve out the growth rate of GDP? $\Delta Y/Y = \frac{s}{c}$

The Harrod-Domar Model

$$Y = \frac{\Delta K}{s}$$

GDP depends on K

- As a result,

$$S = sY = c\Delta Y = \Delta K = I \quad (3.5)$$

- From $sY = c\Delta Y \quad (3.6)$

We derive the rate of growth of GDP

$$\frac{\Delta Y}{Y} = \frac{s}{c} \quad (3.7)$$

- Suppose $c=3$, if $s=6\%$, what is the growth rate? What if $s=15\%$? How to get a growth rate of 7% ? Is it feasible in reality?

可能的

The Harrod-Domar Model

- In the absence of government, the growth rate of national income will be directly related to the savings ratio, and inversely related to the economy's capital-output ratio.

$$g = \frac{\Delta Y}{Y} = \frac{s}{c} \quad g \propto s, \quad g \propto c^{-1}$$

- Furthermore, consider more about c
 - That is the amount of capital needed for one unit of increase of GDP c : 资本产出比
 - c is determined by two things
 - Capital intensity (eg. steel vs. apparel)
 - Efficiency (idle machines)

Capital-output Ratios (c)

Country	1970-79	1980-89	1990-97
United States	3.4	3.6	2.8
Japan	4.9	4.6	6.5
South Korea	2.7	2.8	3.7
India	3.7	2.8	2.9
Indonesia	2.2	3.2	3.0
Argentina	4.8	9.8	2.0
Brazil	2.2	4.2	3.4
Ivory Coast	2.6	4.3	1.8
Venezuela	5.6	6.7	2.8
Kenya	2.3	3.7	4.0

∧

+

+

✓

∧

∧

∧

∧

∧

+

22

The Harrod-Domar Model

- The fixed coefficient production function *固定系数产出函数*
 - Capital (K) and labor (L) are always used in a fixed proportion *K、L 固定比例*
 - Constant return to scale: when K and L are doubled, Y is doubled
- Example: to produce 100 tons of cement a year, a country needs \$10 million and 100 workers.
 - Q1: how many tons can this country produce when $K = \$20 \text{ m}$, $L = 200$? *200 tons*
 - Q2: how many tons when $K = \$15 \text{ m}$, $L = 200$? *150 tons*
- What does the production function look like?

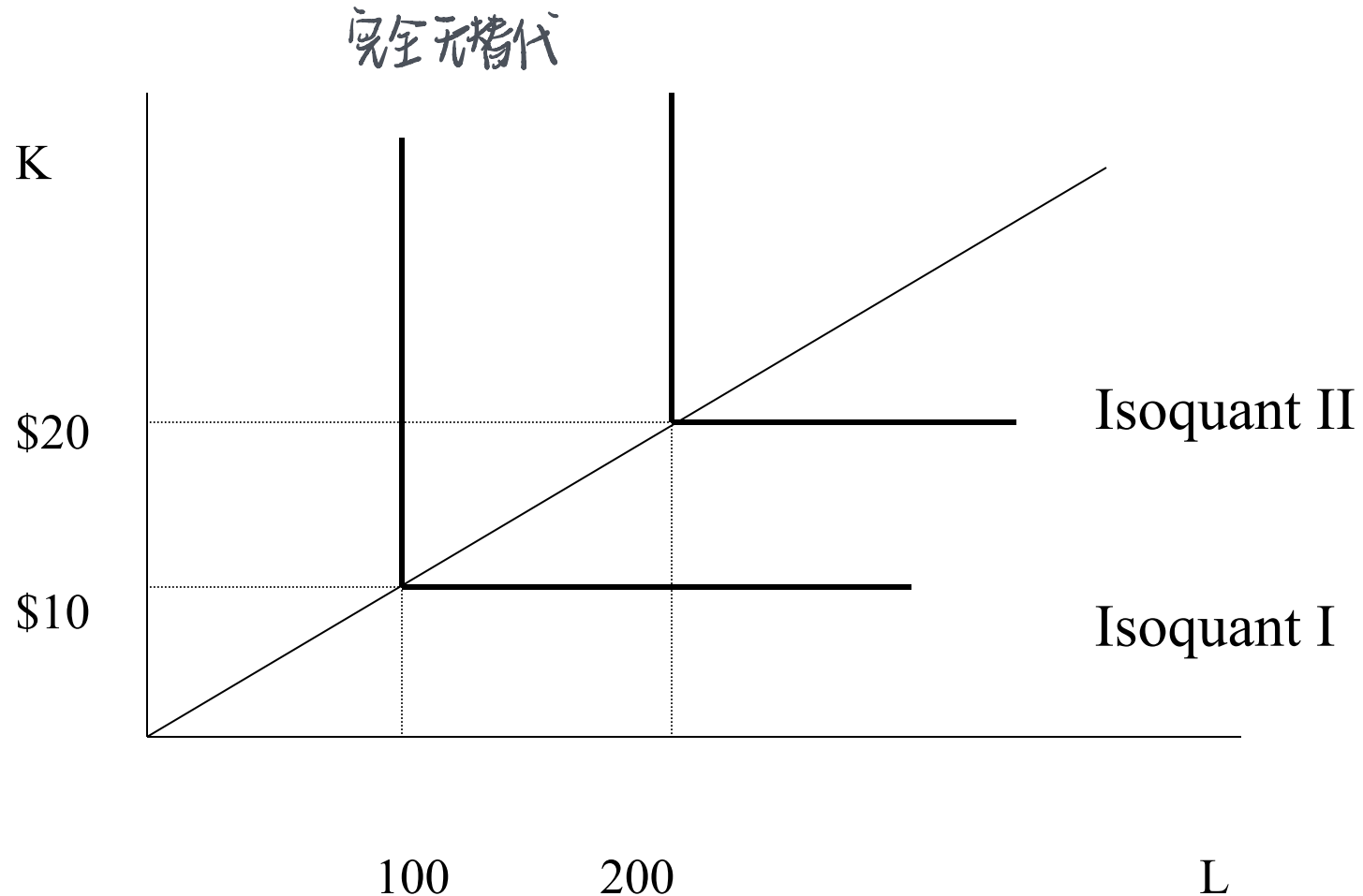
The Harrod-Domar Model

- What does the production function look like?

$$Y = \min\{10K, L\} \quad \text{Leotiff}$$

- Can you draw an isoquant (equal-production) curve? 等量曲线

The fixed-coefficient production function



Criticisms of the Stages Model

- Not suitable for long-term predictions, since c changes over time 随时间改变
- No technological change (constant K/Y ratio)
- Fixed K/L ratio: no substitution between labor or capital. (Hiring one more worker cannot increase production without buying one more machine.)
 - K and L have to grow at the same rate; otherwise, there will be unemployment of either resource (fall off the **knife-edge**)
- Assume the existence of social and institutional structures necessary for production and growth 制度完备
- Assume a country can determine its savings and investment

3.3 Structural-Change Models

- The transformation of an agricultural economy to a more industrialized one
- Employ the tools of neoclassical price and resource allocation theory and econometrics 价格中性、资源分配理论、计量
- W. Arthur Lewis' "two-sector surplus labor" theory
- Hollis B. Chenery and his coauthors' "patterns of development" empirical analysis



Nobel Laureate of Saint Lucia
Sir William Arthur Lewis

Sir William Arthur Lewis (23 January 1915 – 15 June 1991) was a [Saint Lucian](#) economist well known for his contributions in the field of [economic development](#). In 1979 he won the [Nobel Memorial Prize in Economics](#).

The Lewis Two-sector Model

劳动力过剩

- Apply to surplus-labor developing nations around 1970s and also nowadays China
- Two sectors: agriculture (surplus labor) and industry
- Labor transfers to modern sector
- Modern sector grows, which drives economic growth
- Wage rate in urban sector is constant: a little higher than the subsistence wage in rural areas

i.e. $\left(\frac{W}{L}\right)_{\text{urban}} = u > \left(\frac{W}{L}\right)_{\text{rural}} = r$

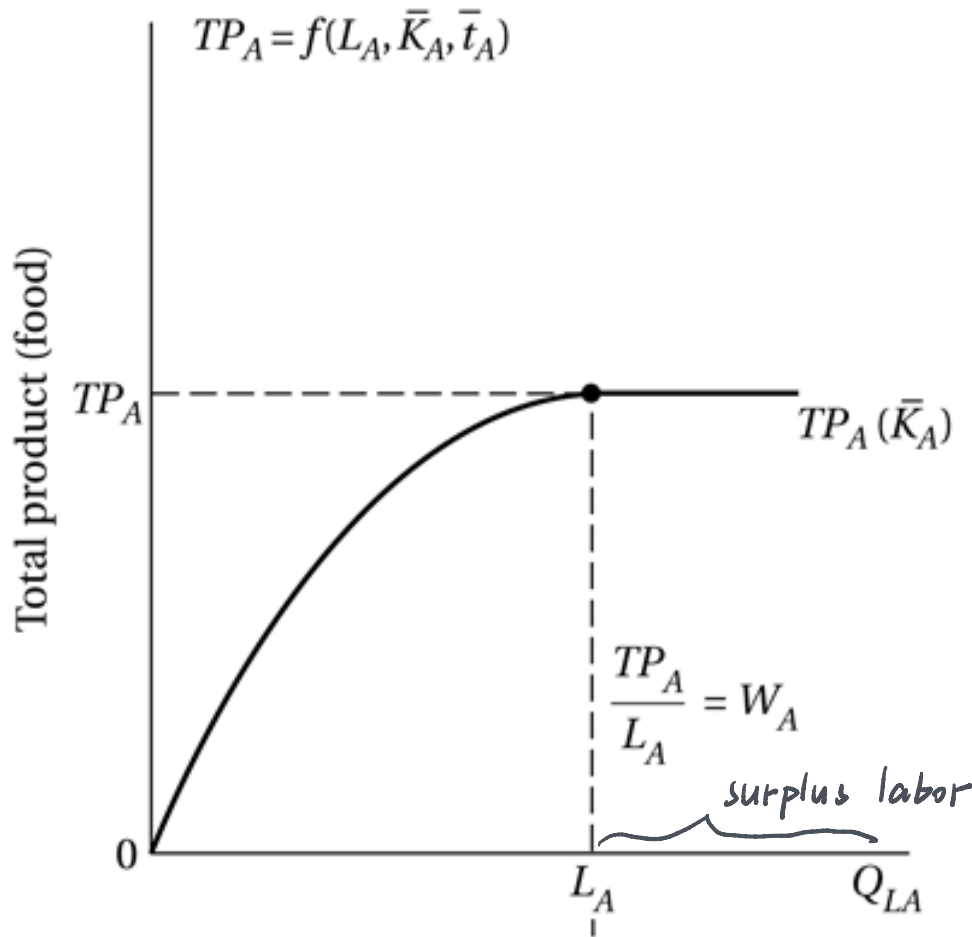
Agricultural Sector

$$\bar{K}_A, \bar{t}_A, W_A = \frac{TP_A}{L_A}$$

- Fixed capital K_A and unchanging technology t_A
- Production only varies with labor input L_A
 - Decreasing return to scale in terms of labor 边际收益递减
 - Zero return to labor after certain point
(**surplus labor**: production does not increase with extra labor)
 - All workers share equally of the production

$$W_A = TP_A / L_A$$

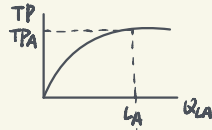
Total agricultural product



$$W_A = \frac{TP_A}{L_A}$$

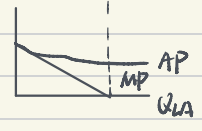
$$W_{AL} = \frac{TP_A L_A - TP_A}{L_A^2}$$

$$= \frac{MP_A \cdot L_A - TP_A}{L_A^2}$$



$$w_A = \frac{TP_A}{L_A}$$

拐点前 $w'_A = \frac{MP_A \cdot L_A - TP_A}{L_A^2} > 0$



$$MP_A \cdot L_A > TP_A$$

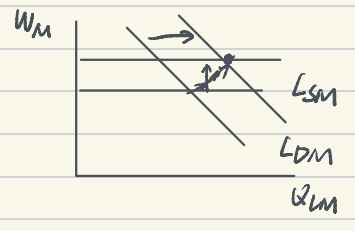
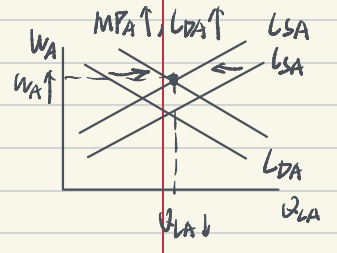
$$L_{SA} = L_{DA} = MP_A > \frac{f(L_A)}{L_A}$$

L_{DA}, MP_A

$$w_M \uparrow = \overbrace{MP_{LM}} = L_{DM} \uparrow$$

$$L_D = L_{DA} \uparrow + L_{DM} \quad \bar{L}_S = L_{SM} \uparrow + L_{SA} \downarrow$$

$$L_D = MP_A + L_{DM} = MP_{LM}$$



$$w_M = MP_{LM}$$

$L_{SM} \uparrow, L_{DM} \uparrow, Q_{LM} \uparrow$

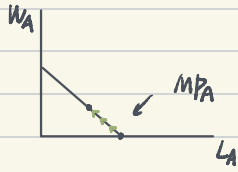
要素供给原则 $MP_L = L_D$

$W_A < W_M$

$TP_A = f(L_A)$

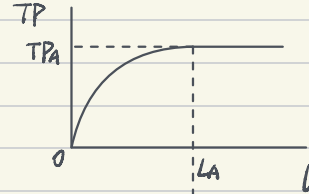
$MP_A = f'(L_A) > 0$

$MP_A' < 0$



$$W = MP_L = MP_M + MP_A = 0$$

农业部门 TP、AP、MP修正。



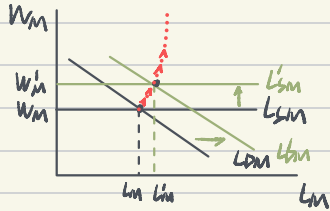
劳动力总量一定

$L = L_A + L_M$ $L_A \downarrow, L_M \uparrow$

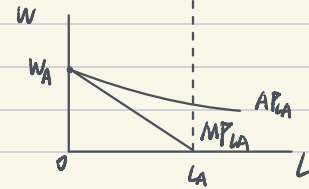
即 $L_M \uparrow$ (劳动力数量即为供给, 无失业)

$L_M = L_{SM}$, 故 $L_M \uparrow$

$L_M = MP_{LM}$. $MP_{LM} \uparrow$



$$W = MP_{LM} + MP_A \uparrow$$



为什么不沿 L_{SM} 向右移动? 而是 L_{SM} 向上移动到 L_{SM}'

$W_M = MP_{LM}$ 而 $W_M' = MP_{LM}' < 0$

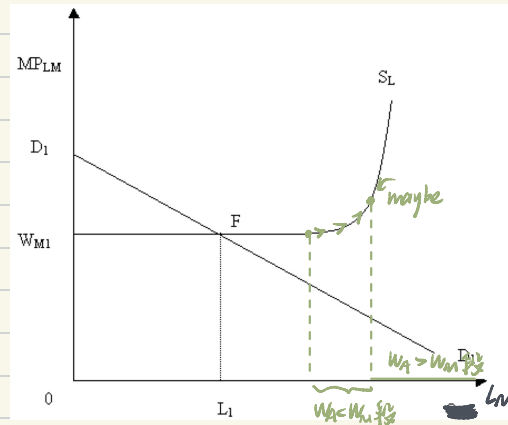
要素边际收益递减

$\Delta L_M = 1$, 有 $0 < \Delta W_M = \Delta MP_{LM} < 1$

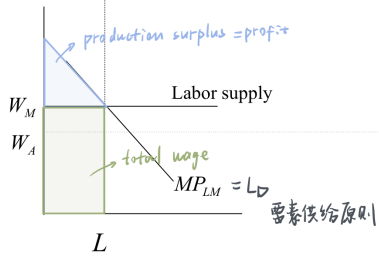
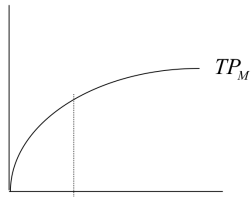
故不会向右沿线移动, 而是向上移动。

此时仍假设 L_{SM} 水平, 因为 $W_A < W_M$, 劳动力无限供给, 不矛盾。

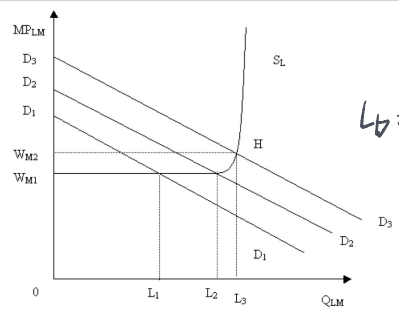
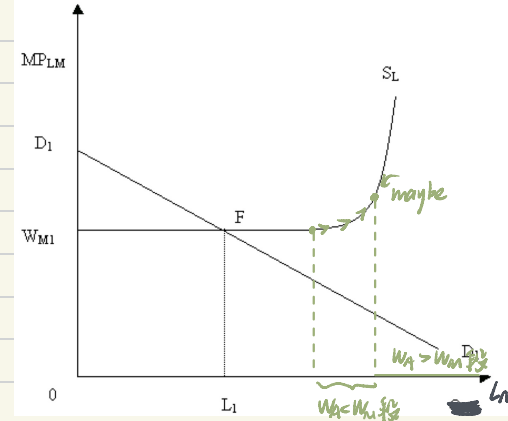
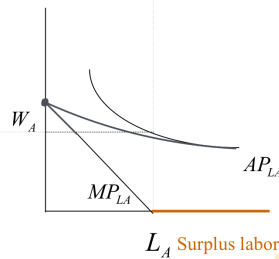
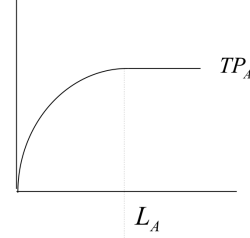
当 $W_A = W_M$ 时, 第一转折点, L_{SM} 倾斜, 会使 $MP_{LM} - L_M$ 更倾斜。



Industrial (modern) sector



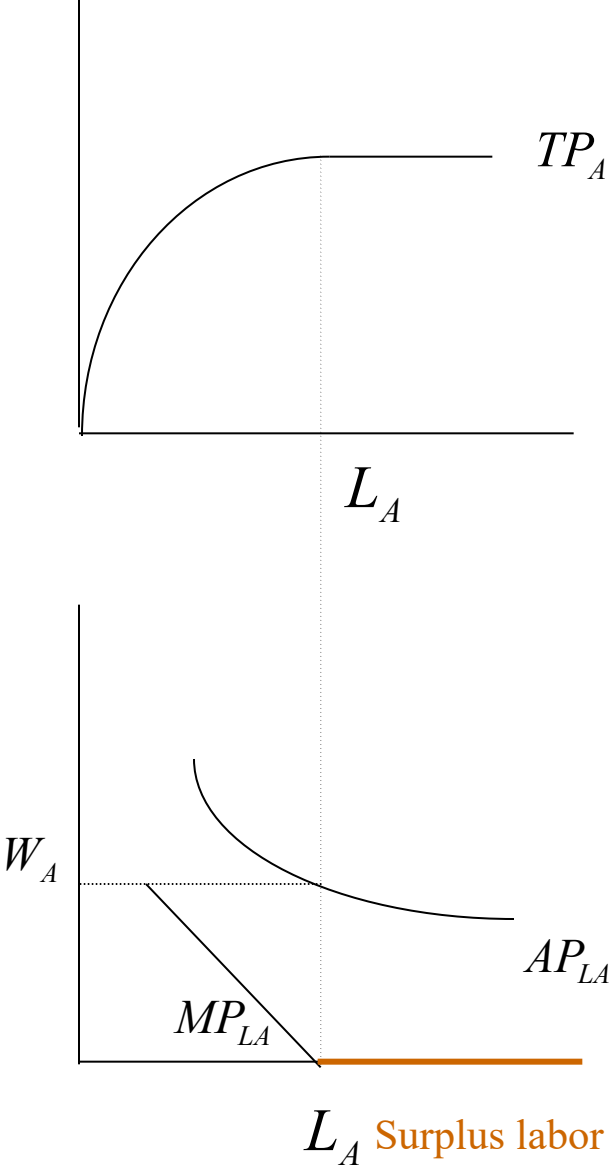
Agricultural sector



$L_D = MP_{LM}$

Agricultural sector

1. Draw a production function, which becomes flat at certain point
2. From production function, we get the MP curve, or the labor demand curve; MP curve turns zero at the same point where TP curve becomes flat
3. There is no labor supply curve
4. Employment is up to the point where MP becomes zero; wage is determined by the average product, where there is no labor market (subsistence economy)

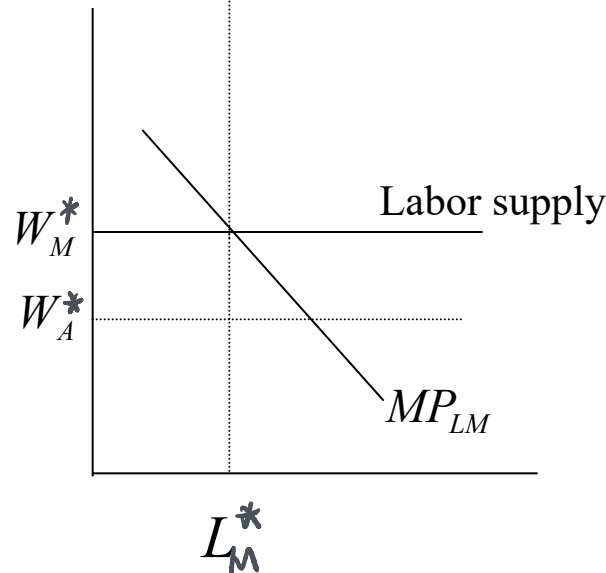
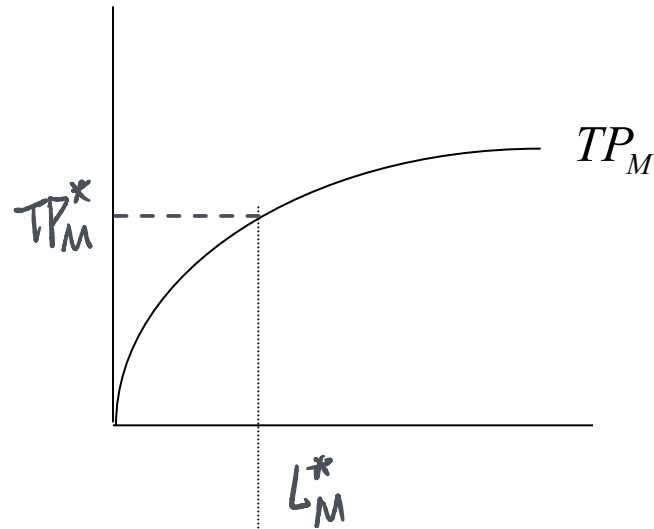


Agricultural labor

Industrial (Modern) Sector

- Fixed capital \bar{K}_M , and technology \bar{t}_M
- Production TP_M varies with labor input L_M
 - Decreasing return to scale in terms of labor 规模不经济
 - Decreasing marginal product of labor (labor demand curve) 劳动的边际产品递减
- Perfectly elastic labor supply curve (unlimited supply of labor from rural areas) 完全弹性的 S_L

Industrial (modern) sector



$W_A^* < W_M^*$
↑
migration
of labor

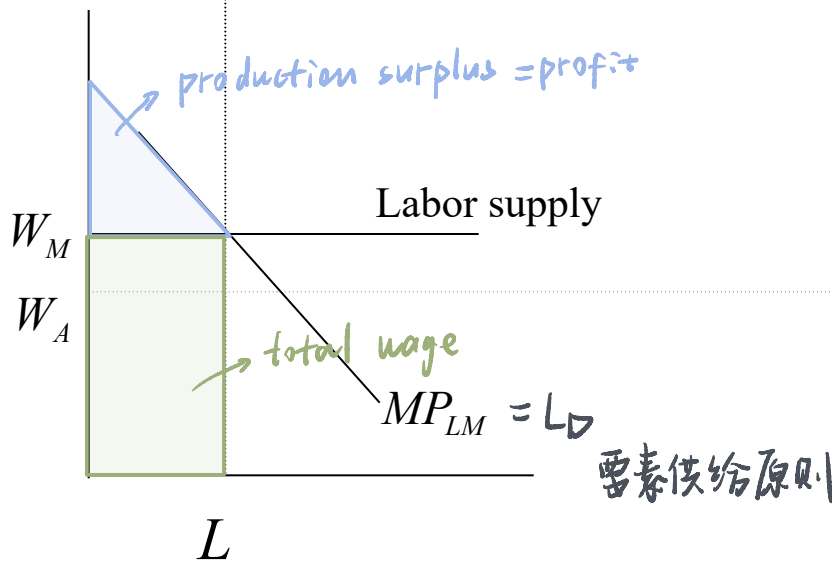
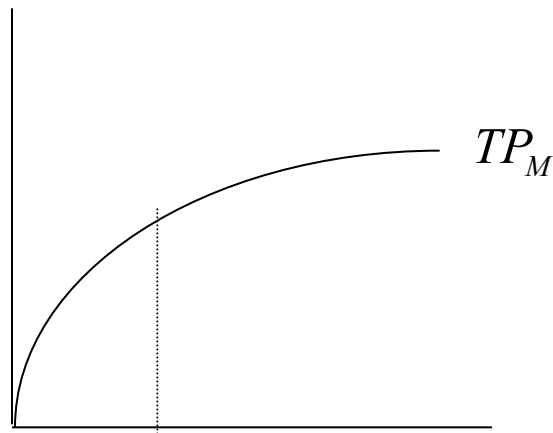
1. Draw a production function
2. From production function, we get the MP curve, or the labor demand curve
3. Labor supply is horizontal, and wage in the modern sector is higher than agricultural sector
4. Labor supply and labor demand determine the wage level and employment in industrial sector: L and W_M



The Lewis Two-sector Model

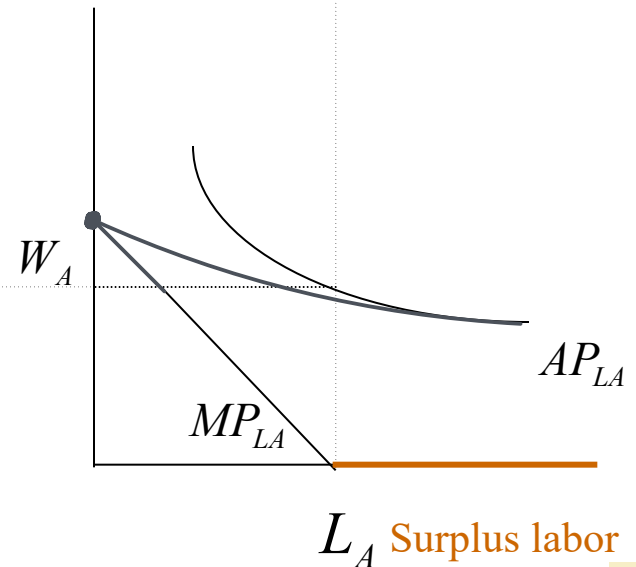
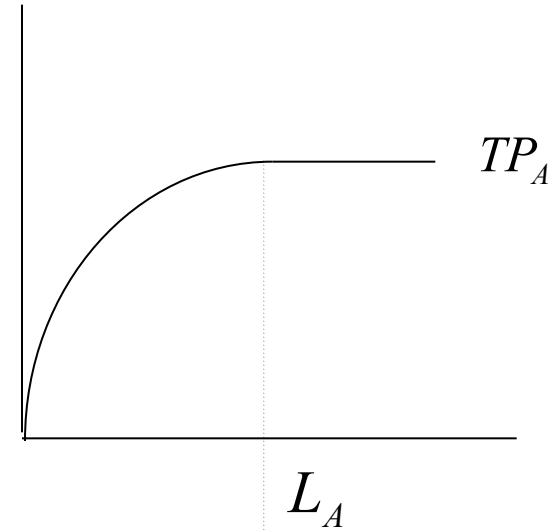
- Two sectors together
 - Migration
 - Industrialization

Industrial (modern) sector



Industrial labor

Agricultural sector



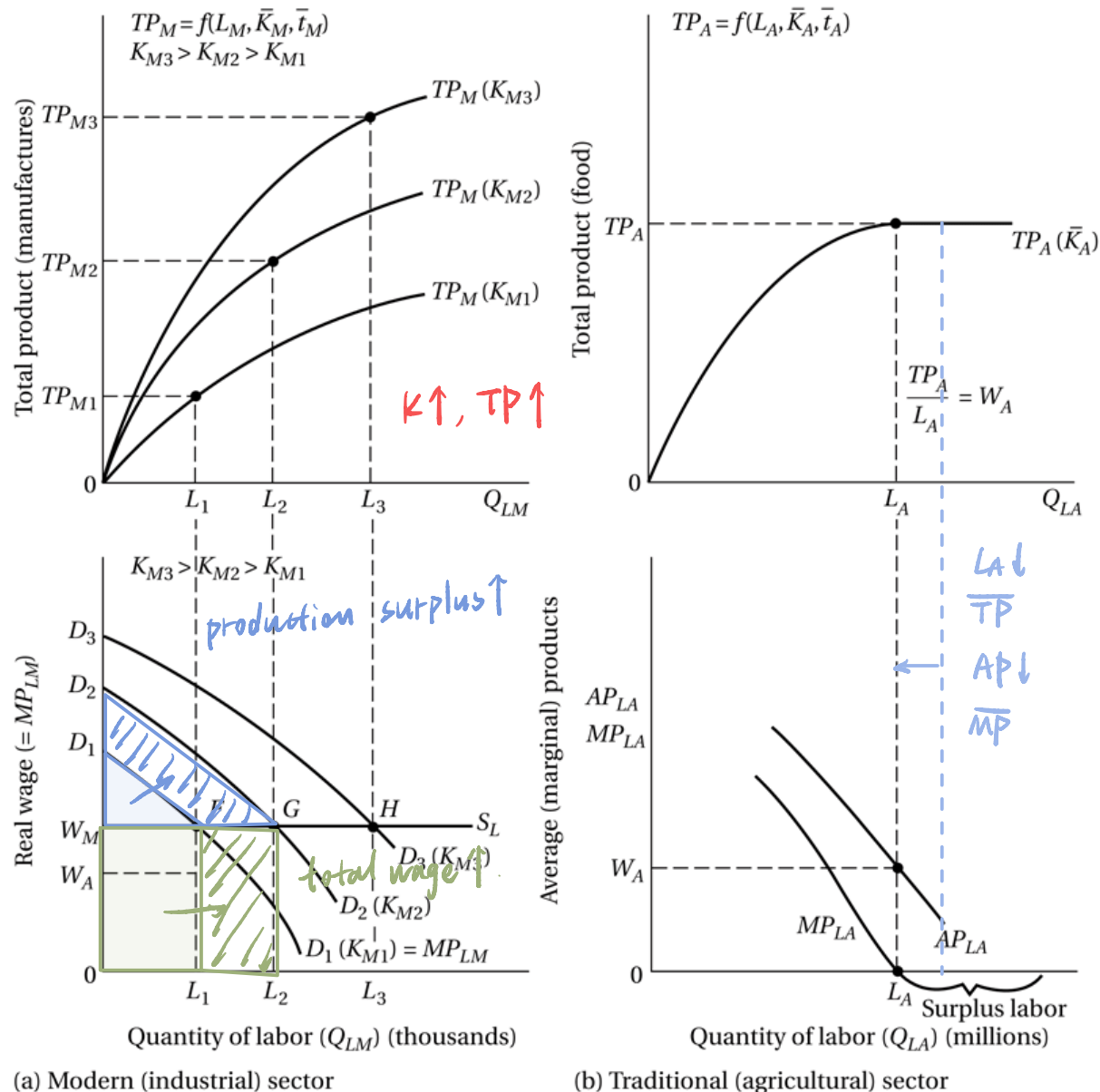
Agricultural labor

The Lewis Two-sector Model

- Any profit in the agricultural sector
 - No
 - All products are shared by labors; and consumed
 - So, no investment
- Further assumptions of the model
 - Capitalists in the modern sector make a lot of profits because of cheap labor (Which part on the graph?)
 - They reinvest their money
 - Capital will grow from K_{M1} to K_{M2} to K_{M3} $\bar{K}_{M1} \rightarrow \bar{K}_{M2} \rightarrow \bar{K}_{M3}$ 扩大生产
 - Production function will shift up
 - As a result, more rural labors are absorbed in industries, and the economy will grow

Figure 3.1

The Lewis Model of Modern-Sector Growth in a Two-Sector Surplus-Labor Economy



The Lewis Two-sector Model

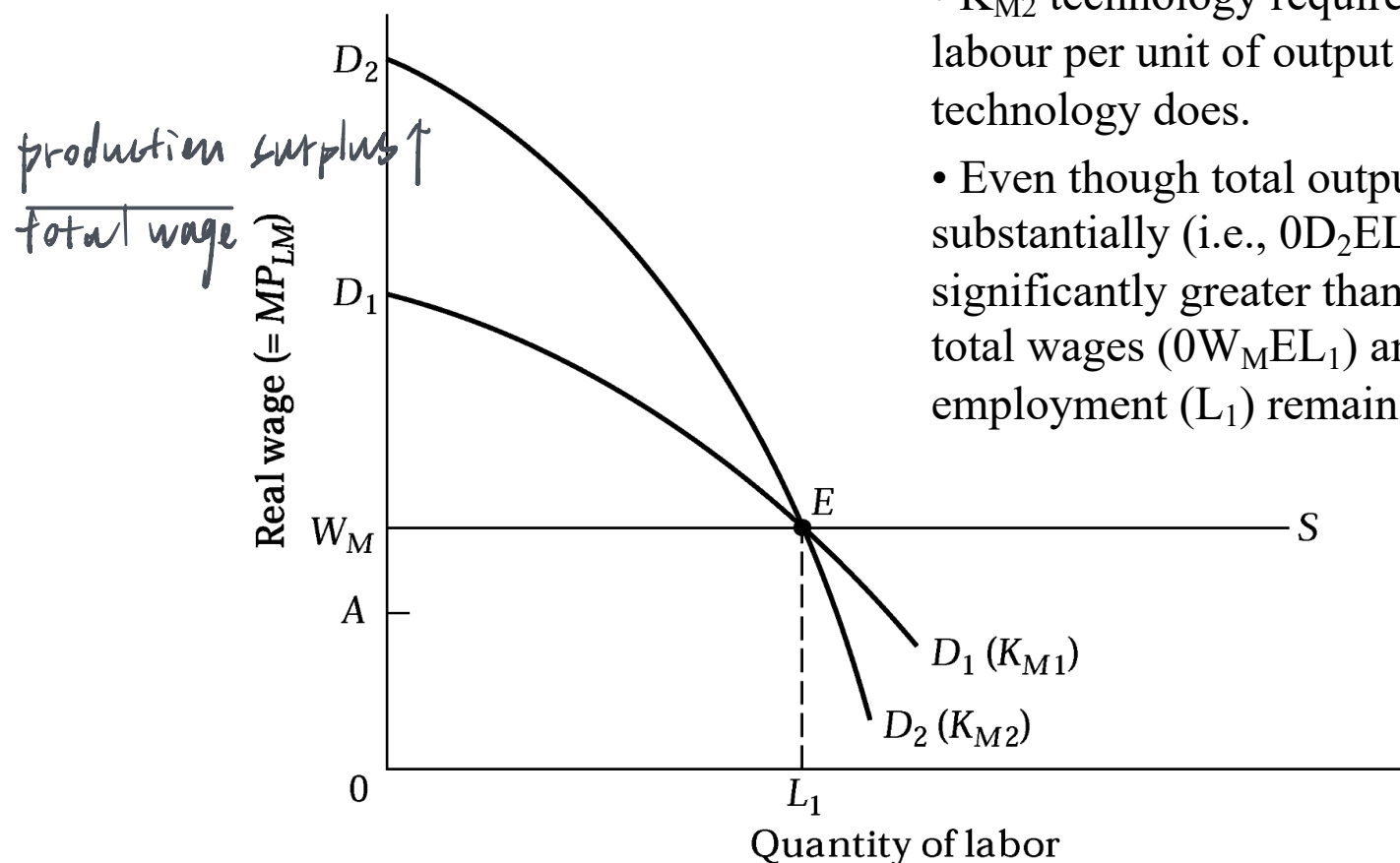
- This ***self-sustaining growth*** will not last forever
- When all surplus labor is absorbed by the industrial sector, further withdraw of labor from agriculture will
 - reduce agricultural production
 - Increase marginal product of labor in the agricultural sector (more land per capita) to a level above zero
 - No cheap labor anymore; to attract more rural labor, the industrial sector has to increase wage level. This implies an upward sloping labor supply curve

Criticisms of The Lewis Model

- The model assumes surplus labor in rural areas but full employment in urban areas; this is not true in reality (China?) *unemployment* → *Todaro Model*
- The model assumes a constant wage in urban sector until all surplus labor is absorbed; not true in reality either (union, multinational corporations tend to raise urban salaries substantially) *工会、跨国公司*
- The model assumes that capitalists just duplicate production when they make money; but they could also invest in some labor-saving **technologies**, which will cause **antidevelopment growth** (see next figure)

科技提升, 节约劳动以吸收更少劳动力

Figure 3.2 The Lewis Model Modified by Laborsaving Capital Accumulation:
Employment Implications



- K_{M2} technology requires much less labour per unit of output than K_{M1} technology does.
- Even though total output has grown substantially (i.e., $0D_2EL_1$) is significantly greater than $0D_1EL_1$), total wages ($0W_MEL_1$) and employment (L_1) remain unchanged.

All extra income and output growth is distributed to the few owners of capital, while income and employment levels for the masses of workers remain largely unchanged.

The Lewis Two-sector Model

- Conclusions
 - We studied the Lewis model
 - It describes the process of labor transfer from agriculture to industry and growth driven by the expansion of industries

3.4 The International-Dependence Revolution 国际依赖革命

- The neocolonial 新殖民主义 dependence model
 - It is an indirect outgrowth of Marxist thinking
 - Legacy of colonialism 殖民遗产
 - Unequal power
 - Core (the developed) – periphery (外围 the developing)
- The false-paradigm model 虚假范例模型
 - Pitfalls of using “expert” foreign advisors who misapply developed-country models
外国专家提供了不适应国情的建议

3.4 The International-Dependence Revolution

- The dualistic-development thesis 二元发展论
 - Superior and inferior elements can coexist
 - The coexistence is chronic 长期的 优越/劣等长期共存
 - The degrees of superiority 优势 or inferiority 劣等 tend to increase
 - The superiority elements does little or nothing to pull up the inferior elements
- Criticisms and limitations
 - Does little to show how to achieve development in a positive sense
 - Accumulating counterexamples (e.g. China, India)

3.5 The Neoclassical Counterrevolution 新古典主义革命: Market Fundamentalism

市场经济基本原则

- Challenging the Statist 计划经济 Model: Free Markets, Public Choice, and Market-Friendly Approaches
 - Free market approach 自由市场分析: markets alone are efficient
 - Public choice approach 公共选择理论: government does nothing right
 - Market-friendly approach 亲善市场理论: admit market failure
- Main Arguments
 - Denies efficiency of intervention 否认干预的有效
 - Points up state owned enterprise failures } 政府和国有企业无效率
 - Stresses government failures
 - Traditional neoclassical growth theory - with diminishing returns, cannot sustain growth by capital accumulation alone 资本积累不能带来持续增长

回报递减

Solow → Ramsey → Overlapping Generation → Endogenous Growth
5

- The Solow 索洛 Growth Model:
 - Builds on the production model by adding a theory of capital accumulation 积累
 - Was developed in the mid-1950s by Robert Solow of MIT
 - Was the basis for the Nobel Prize he received in 1987



Robert Merton Solow is an American economist, particularly known for his work on the theory of economic growth that culminated in the exogenous growth model named after him. [Wikipedia](#)

Born: August 23, 1924 (age 92 years), [Brooklyn, New York City, NY](#)

Education: [Harvard University](#)

Doctoral advisor: [Wassily Leontief](#)

- Additions / differences with the model:
 - Capital stock is no longer exogenous 外生.
 - Capital stock is now “endogenized 内生.”
 - The accumulation of capital is a possible engine of long-run economic growth.

The Solow Model: Assumptions

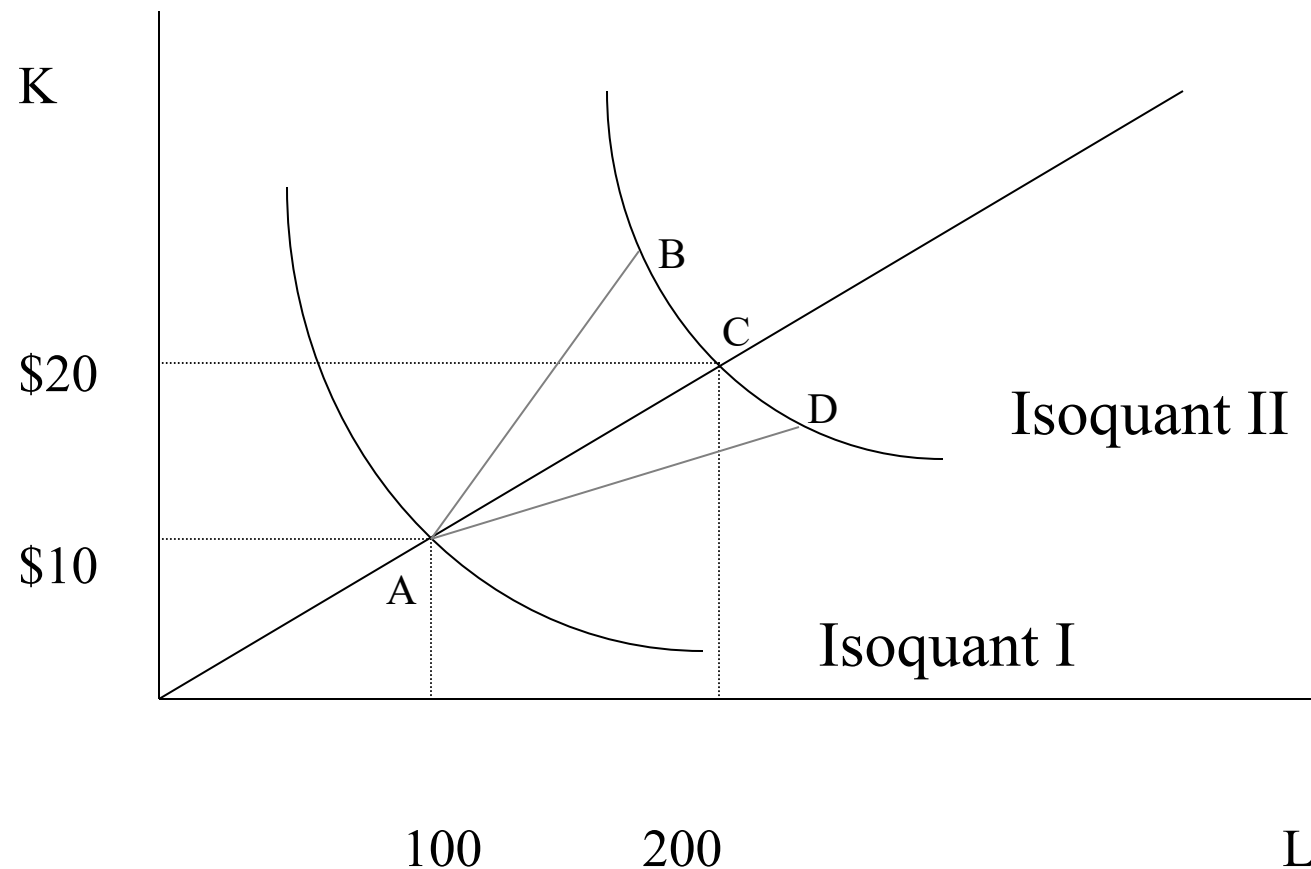
- The neo-classical production function
 - K/L and K/Y are not fixed anymore
 - They vary, depending on the endowment of the economy
 - Decreasing returns to capital
 - Allows substitution between labor and capital
- Example?
Cobb-Douglas production function

- Cobb-Douglas production function

$$Y = AK^{0.5}L^{0.5}$$

- Most popular function form for production
 - Assume A is determined exogenously
 - Decreasing return to capital
- Example:
 - $A=1, K=10, L=100$, then $Y=?$
 - When K is doubled, then $Y=?$
 - Is Y doubled?
- Can you draw an isoquant curve?

The neo-classical production function



- 3 ways to expand the economy
 - AC: same proportion of K and L
 - AB: labor-saving expansion
 - AD: capital-saving expansion

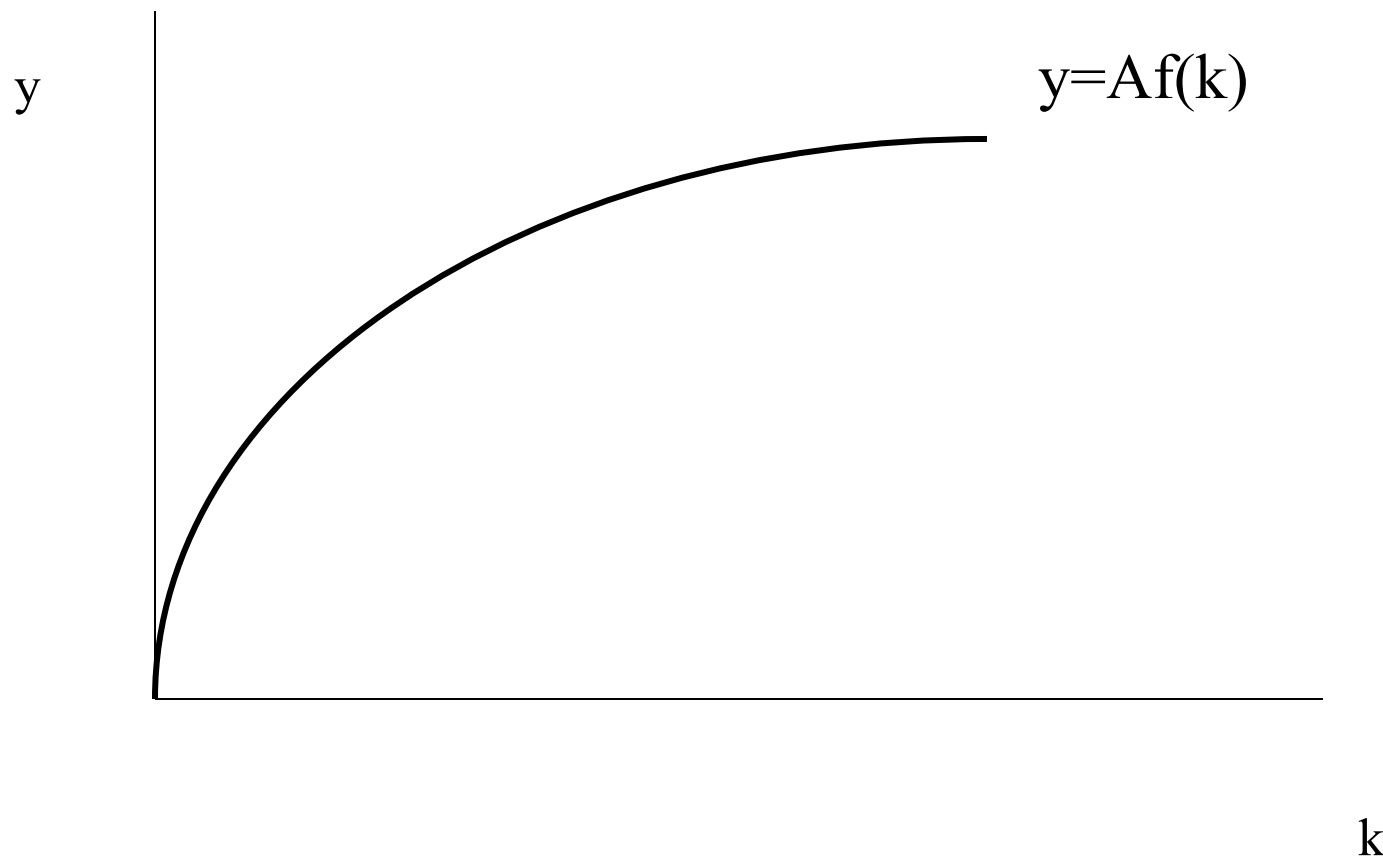
- Divide both sides by L *y*: output per worker

$$y \equiv Y / L = (AK^{0.5} L^{0.5}) / L = Ak^{0.5}$$

where $k=K/L$

- A general function form: $y=Af(k)$
- Although Y has a CRTS with respect to K and L , y has DRTS with respect to k . Why?
 - Because k is the number of machines per worker; when k is small, increasing machines will increase productivity a lot; but when k becomes larger, increasing machines will increase productivity less
- Can you draw a production function: y as a function of k ?

The neo-classical production function



The Solow Model

① Production Function

- First equation of Solow model: $y = Af(k)$
- The C-D production function shows that per capita GDP, y , increases in the capital-labor ratio, k
- But, what determines the change of k ?
- By definition, $\Delta K = I - dK$
- If all savings go to investment, $S = sY = I$
- We define $k = K/L$, with total differentiation

$$\Delta k = \frac{\Delta K}{L} - \frac{\Delta L \cdot K}{L^2} = \frac{\Delta K}{L} - \frac{K}{L} \cdot \frac{\Delta L}{L} = \frac{\Delta K}{L} - k \cdot n$$

$$\Delta k = \frac{s \cdot Y - d \cdot K}{L} - k \cdot n = sy - (n + d)k$$

The Solow Model

② Investment Function

- Hence, the second equation of Solow model:

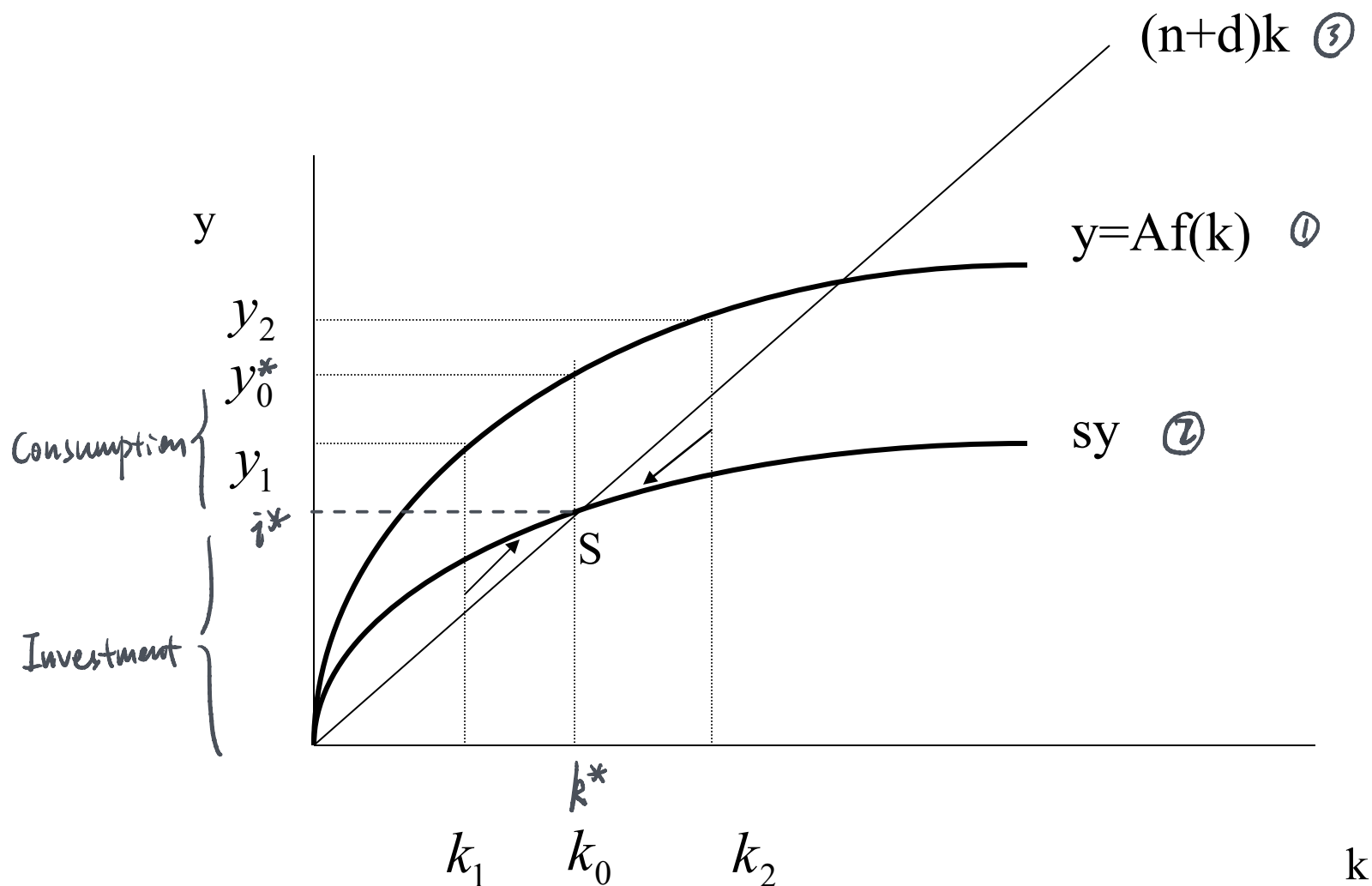
$$\Delta k = sy - (n + d)k$$

- Growth of capital-labor ratio relies on 3 things
 - Savings (s: savings rate)
 - Population growth (n: population growth rate)
 - Depreciation (d: depreciation rate)
- The two equations determine the equilibrium

③ Modified Depreciation

$$(n + \delta)k$$

The Solow Diagram



The Solow Model

- When k is small, for example, $k = k_1$
 $sy > (n+d)k$
 - The amount of saving per person is larger than the amount of saving per person needed to compensate for new workers and depreciation
 - As a result, the amount of capital per person, k , increases; and the economy shifts to the right on the horizontal axis

The Solow Model

- When k is large, for example $k = k_2$
 $sy < (n+d)k$
 - The amount of saving per person is smaller than the amount of saving per person needed to compensate for new workers and depreciation
 - As a result, the amount of capital per person, k , decreases; and the economy shifts to the left on the horizontal axis

The Solow Model: steady state

- When k is at point S , $\dot{k} = 0$
 $sy = (n+d)k$
 - The amount of saving per person is equal to the amount of saving per person needed to compensate for new workers and depreciation
 - As a result, the amount of capital per person, k , will not change; and the economy stays at point S
 - This is called the ***steady state***

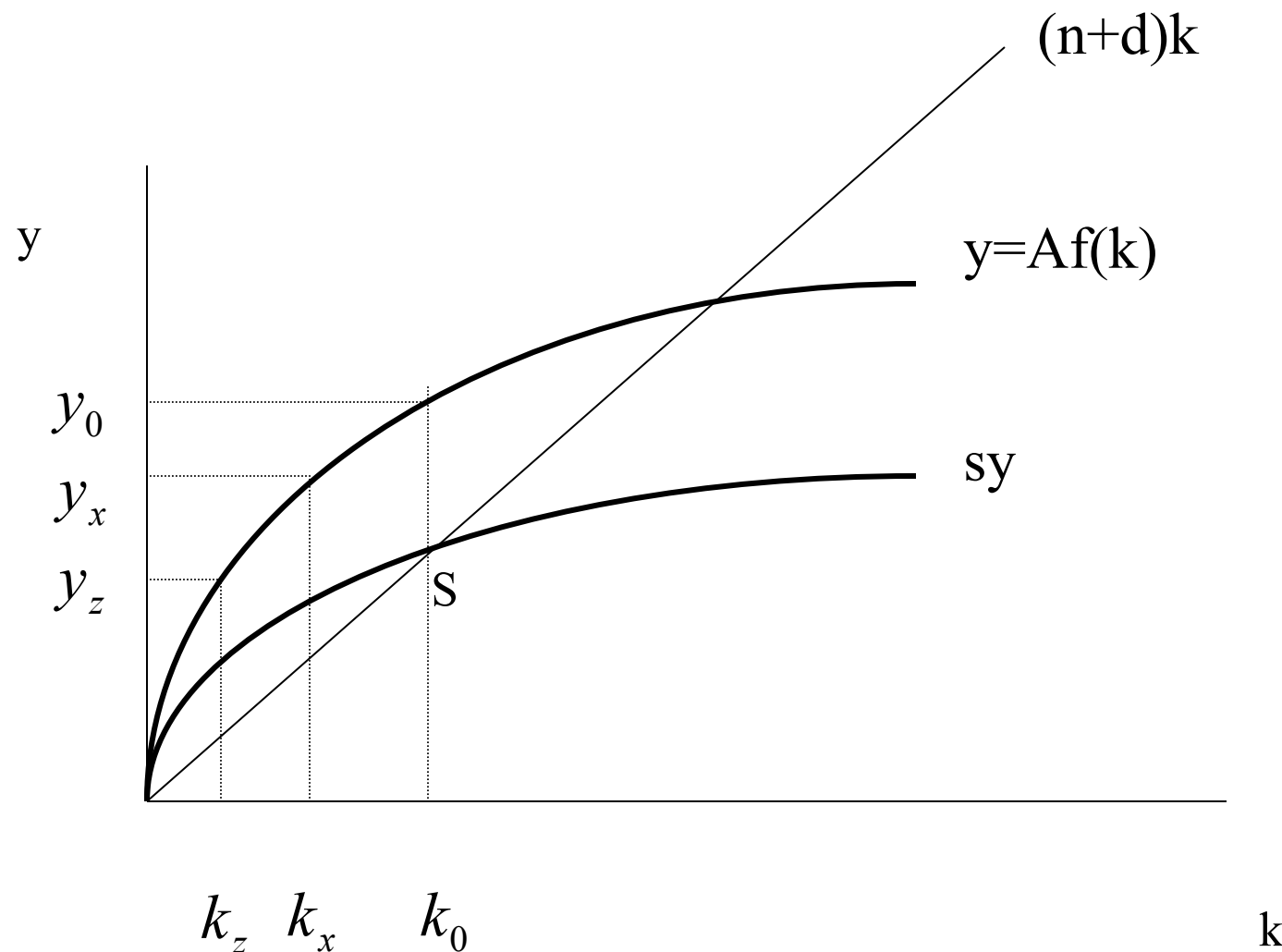
The Solow Model: Steady State

- At the steady state
 - k, y are constant
- Are K and Y also constant?
- Remember that,
 - $Y = yL$ and $K = kL$,
 - and L is growing at a rate of n
- Thus,
 - Y and K are growing at a rate of n
- Example: $y = 10$, L is growing at a rate of 2%, then how fast is Y growing?

The Solow Model: Different initial conditions

- Suppose two countries X and Z
 - have the same A , f , s , n and d
 - but country X has larger k and y than country Z
- Which country grows faster?

The Solow Diagram



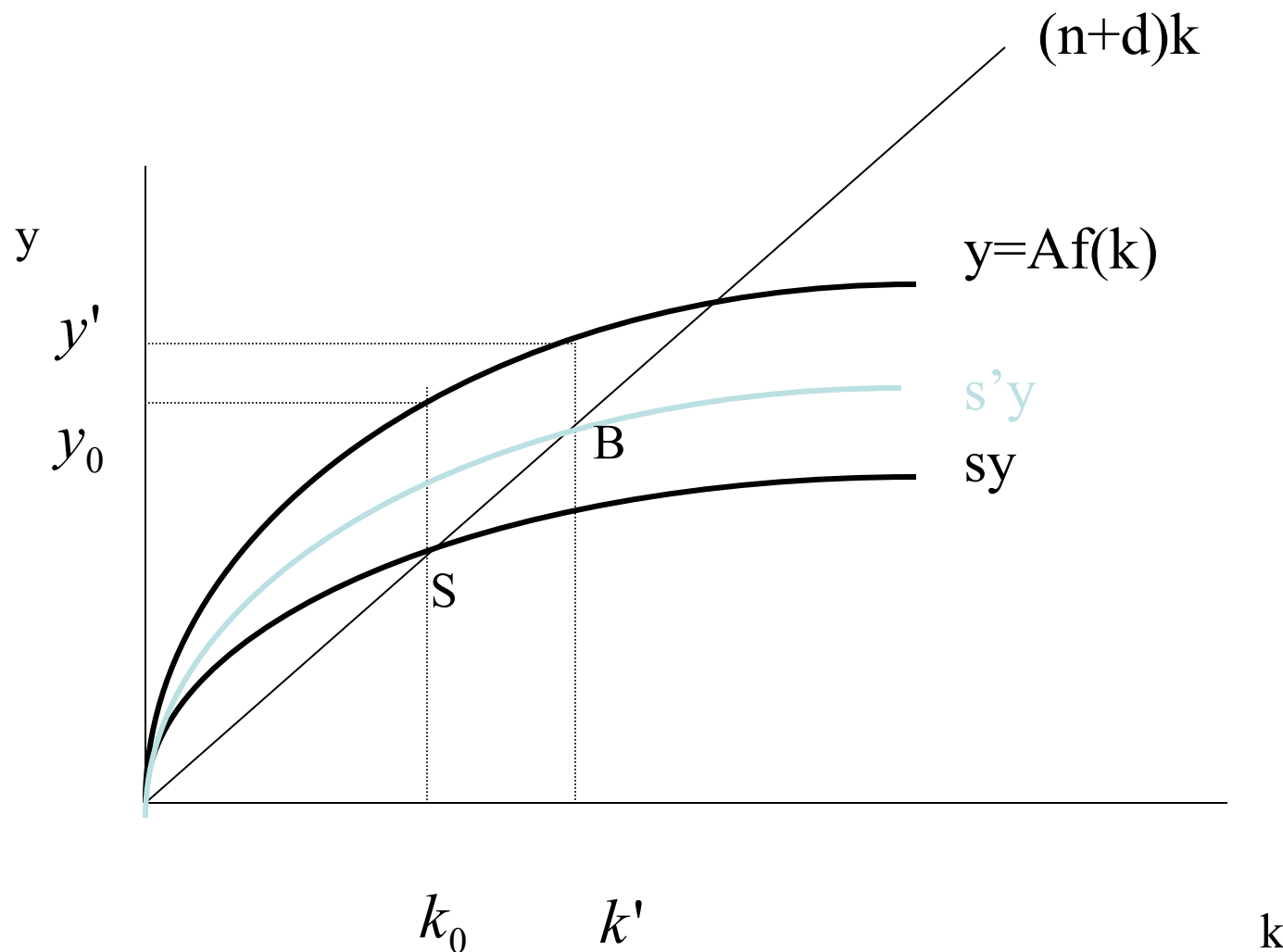
The Solow Model: different initial conditions

- Which country grows faster?
 - Country Z grows faster
 - Because $f(k)$ has DRTS, which means at lower level of k , the slope of $f(k)$ is larger (the same change of k will cause a larger change of y)
- This is the famous **catch-up** theory
 - **Poorer countries grow faster**
 - Poor and rich countries reach the same level of per capital GDP eventually

The Solow Model: how can y grow in the long-run

- Short-run vs. long-run
 - In the short-run, the economy may not be at the steady state, and k and y may grow
 - In the long-run, the economy will reach the steady state, and k and y are stable; however, Y and K grow at the same rate as population grows
- Can a country grow in the long-run?
- And how?

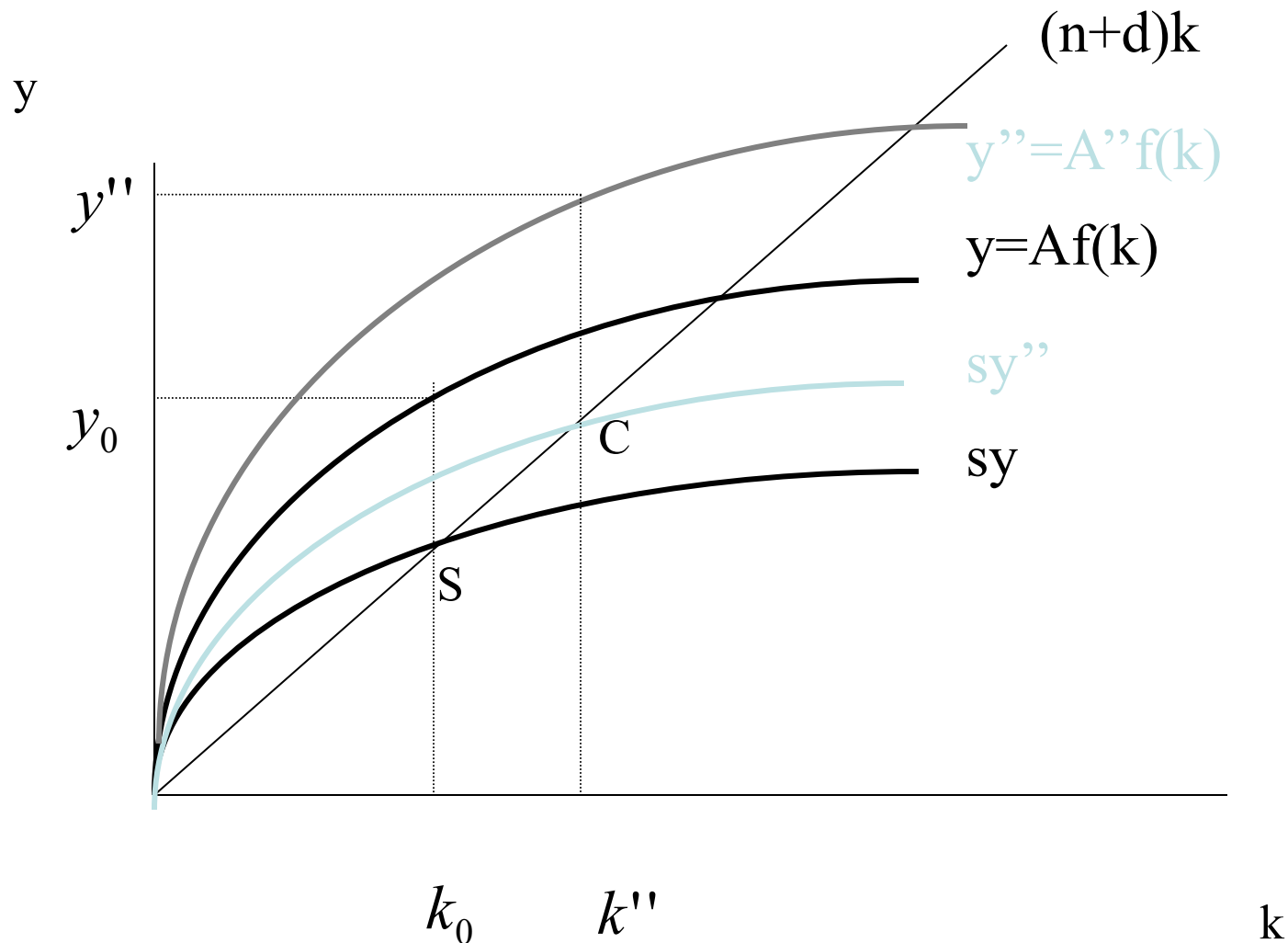
The Solow Diagram: s increases



The Solow Model: s increases

- When s increases
 - Point S is not the steady state anymore
 - Both k and y will grow
 - Until they reach the new steady state, point B
- Although there is a short run growth of k and y (from S to B)
- In the long-run, y and k will be stable; and Y and K will grow at the rate of n

The Solow Diagram: A increases



The Solow Model: technological advances

- When technology improves
 - Point S is not the steady state anymore
 - Both k and y will grow
 - Until they reach the new steady state, point C
- In the long-run, since technology continues to improve and y and k will continue to grow

Strengths and Weaknesses of the Solow Model

- The strengths of the Solow Model:
 - It provides a theory that determines how rich a country is in the long run.
 - long run = steady state
 - The principle of transition dynamics
 - allows for an understanding of differences in growth rates across countries
 - a country further from the steady state will grow faster

- The weaknesses of the Solow Model:
 - It focuses on investment and capital
 - the much more important factor of TFP is still unexplained
 - It does not explain why different countries have different investment and productivity rates.
 - a more complicated model could endogenize the investment rate
 - The model does not provide a theory of sustained long-run economic growth.

- Empirically, however, economies appear to continue to grow over time.
 - Thus, we see a drawback of the model.
- According to the model:
 - Capital accumulation is not the engine of long-run economic growth.
 - After we reach the steady state, there is no long-run growth in output.
 - Saving and investment
 - are beneficial in the short-run
 - do not sustain long-run growth due to diminishing returns

Relevance of growth theories to today's developing countries

- They are elegant models that explain certain aspect of the economic growth
- The biggest problem of all these models: assuming the production process is a black box
- There are many practical questions that are not answered by the model
 - How to make people save, and firms invest
 - How to make investment more efficient
 - Do agricultural workers automatically move to the cities and easily find a job? If not, how to make the structural change happen?
- In fact, each country has its unique problem

The Solow Model: empirical evidence

- Robert Barro, 1991, "Economic growth in a cross section of countries," *Quarterly Journal of Economics*.
 - An academic paper using regression techniques (not required for you to read)
 - Use a group of 98 countries in the period 1960-1985 to study whether income converges
 - Figure 1 of Barro

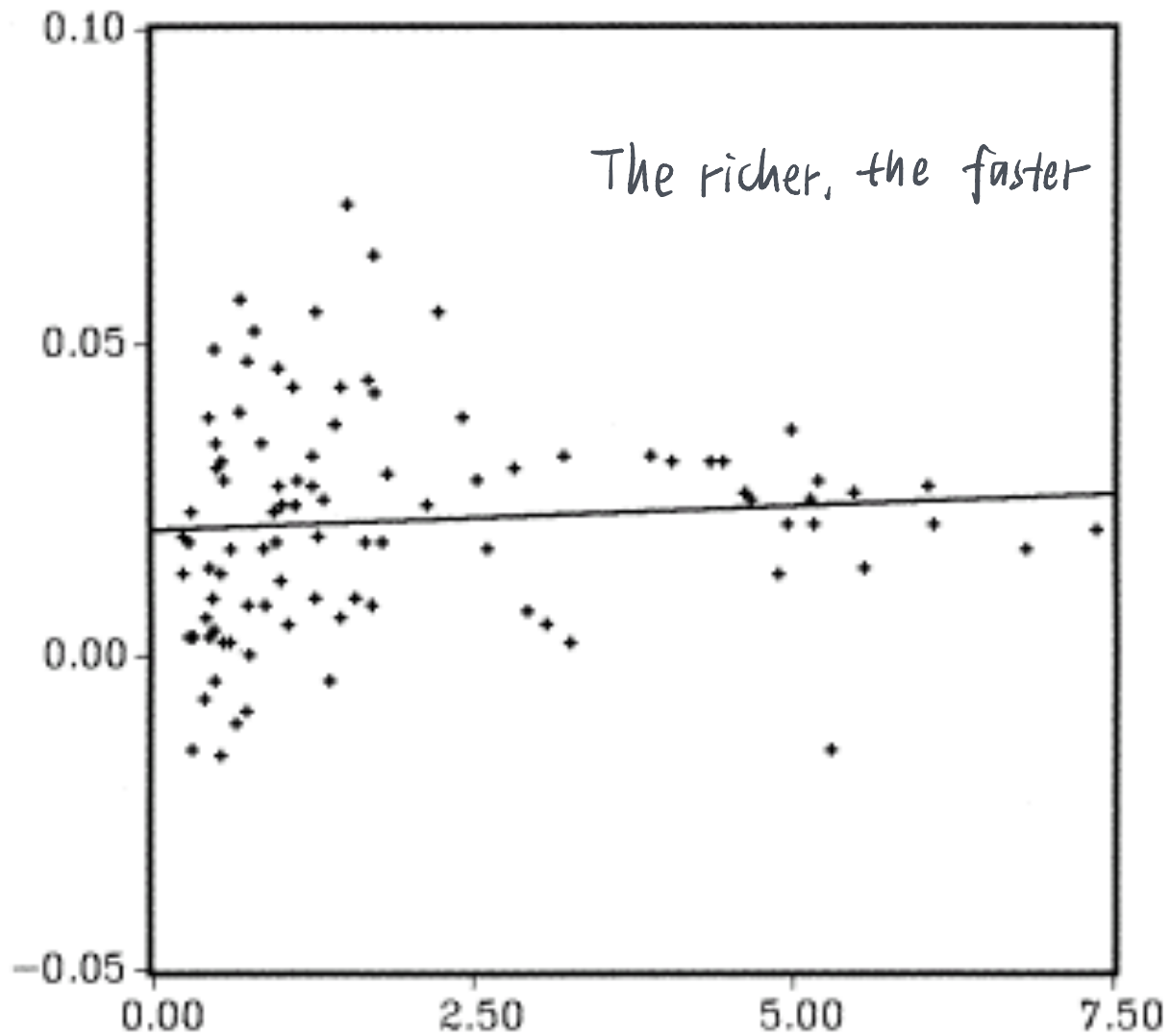


FIGURE I
Per Capita Growth Rate Versus 1960 GDP per Capita

The Solow Model: empirical evidence

- No sign of convergence, why?
- Because
 - Solow model assumes the same steady state for every country
 - There are many other variables that affect growth, such as initial human capital
 - Causality problem in statistics
 - Is it that saving causes growth of GDP, or growth of GDP causes saving?
- After dealing with these issues, Barro found
 - Figure 2 of Barro

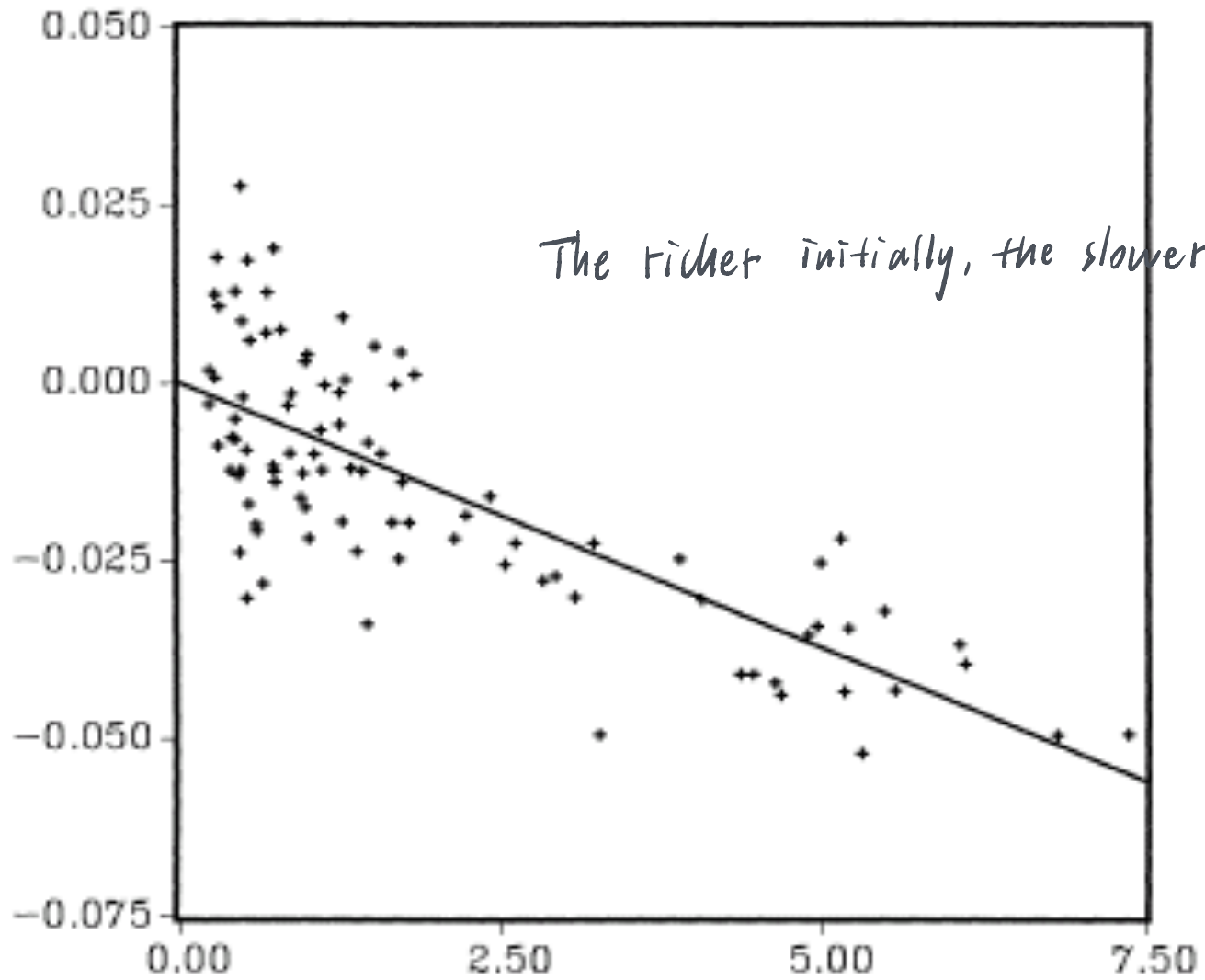
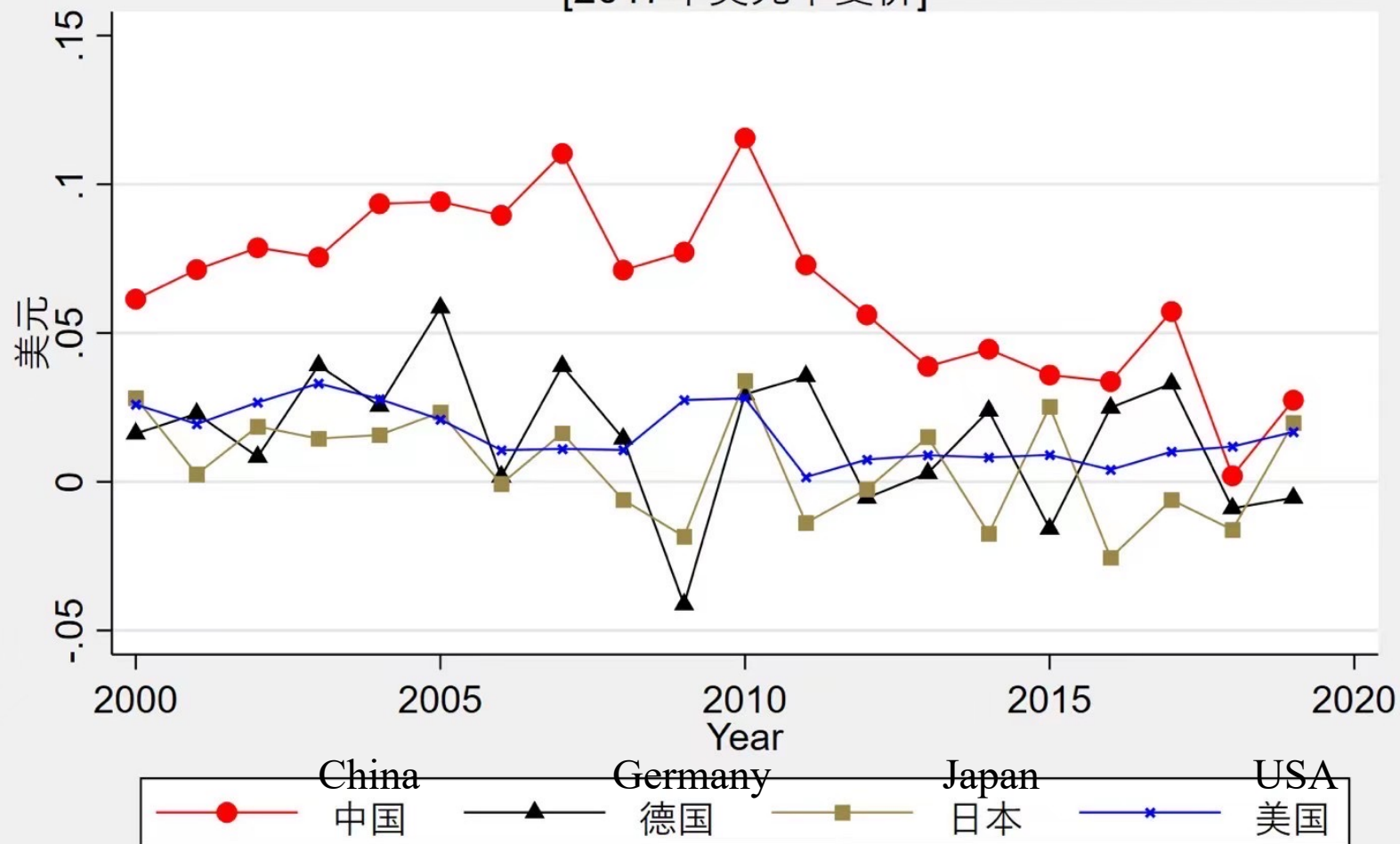


FIGURE II
Partial Association Between per Capita Growth and 1960 GDP per Capita (from regression 1 of Table I)

Growth Rate of Labor Productivity for Major Countries

主要国家的劳动生产率增长率 2000-2019

[2017年美元不变价] Constant USD 2017



数据来源: Pann World Table

3.6 Classic Theories of Development: Reconciling 协调 the Differences

- Governments do fail, but so do markets; a balance is needed
- Must attend to institutional and political realities in developing world
- Development economics has no universally accepted paradigm
- Insights and understandings are continually evolving
- Each theory has some strengths and some weaknesses

Exercise 1. Consider the Harrod-Domar model.

Suppose that initially, a developing country's capital-output ratio (c) is 5, and the savings rate (s) is 12%.

(a) What will be the initial GDP growth rate? $g = \frac{s}{c} = \frac{12\%}{5} = 2.4\%$

(b) Suppose that technological advances cause the capital-output ratio to fall to 4. How will this affect the GDP growth rate? $g' = \frac{s}{c'} = \frac{12\%}{4} = 3\%$

(c) Starting again from the initial situation, suppose instead that the national savings rate is increased to 15%. How will this affect the GDP growth rate? $g'' = \frac{s'}{c} = \frac{15\%}{5} = 3\%$

(d) What does the Harrod-Domar model tell us about the sources of economic growth?

$$g \propto s \quad g \propto c^{-1}$$

g is directly proportional to s
 g is inversely proportional to c

Answers:

a) $g = 12\%/5 = 2.4\%$

b) $g = 12\%/4 = 3\%$

c) $g = 15\%/5 = 3\%$

d) The growth rate is directly related to the saving ratio, and inversely related to capital-output ratio.

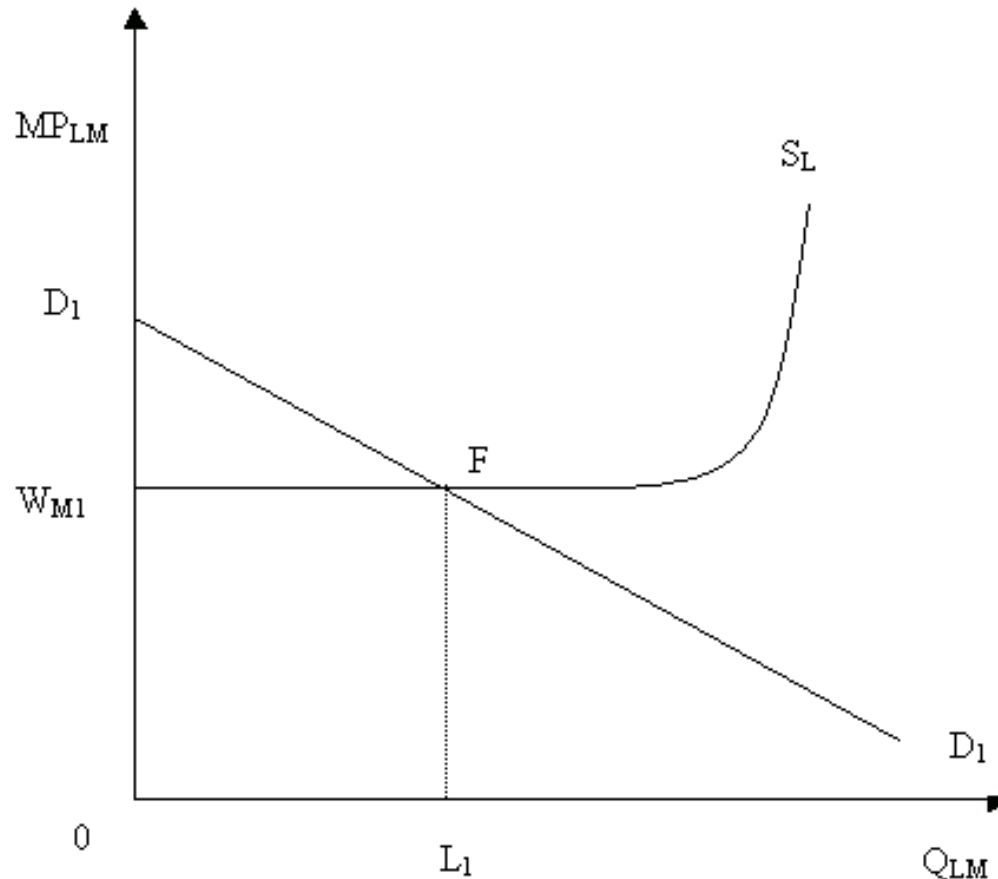
Exercise 2. Lewis Two-Sector Model

Consider Figure 3.1 in the Todaro-Smith textbook. Draw a single graph with real wage in the modern sector (MP_{LM}) on the vertical axis, and quantity of modern sector labor (Q_{LM}) on the horizontal axis. Draw a labor supply curve that is initially perfectly elastic (i.e., horizontal) but that becomes steeply and positively sloped at a certain point. Draw a single downward-sloping labor demand curve that intersect the perfectly elastic portion of the labor supply curve.

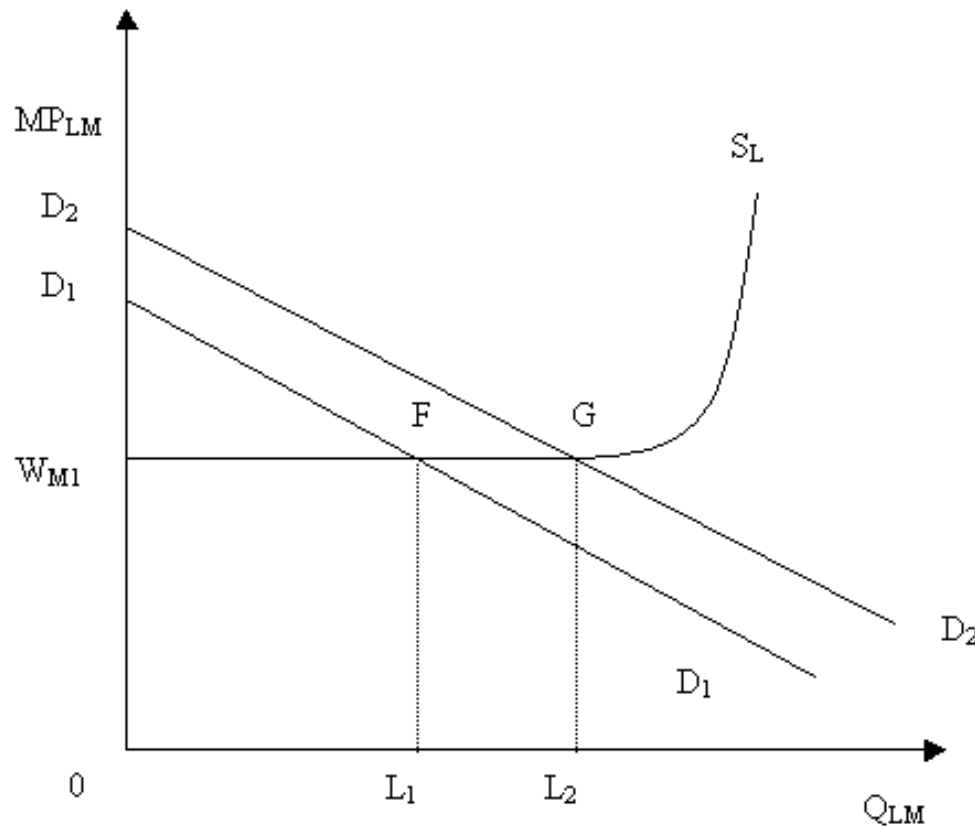
- (a) What area represents total modern sector output? What areas represent share of output paid to laborers in the form of wages and the share of output paid to capitalists?
- (b) Suppose reinvestment of profits causes an increased demand for labor in the modern sector. Draw a new labor demand curve to the right of the original curve, but still intersecting the perfectly elastic portion of the labor supply curve. What happens to total output and to the returns paid to laborers and capitalists?
- (c) Suppose now that further reinvestment of profits causes an increased demand for labor in the modern sector. However, all surplus labor has already migrated to the modern sector, so the new labor demand curve will now intersect the steep portion of the labor supply curve. What happens to total output and to the returns paid to laborers and capitalists?

Answers:

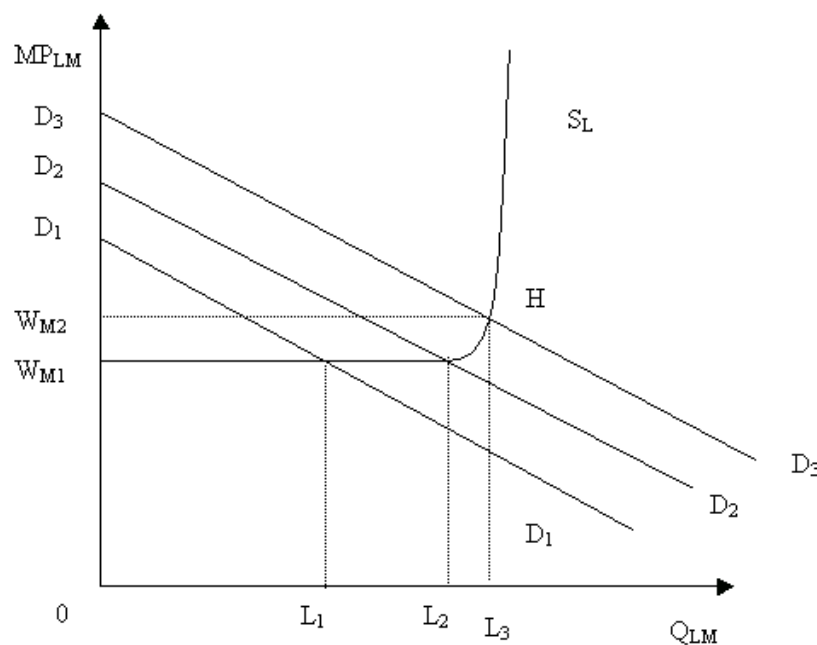
a) Total output is represented by area $0D_1FL_1$. Returns to labor are represented by $0W_{M1}FL_1$, and returns to capital by $W_{M1}D_1F$.



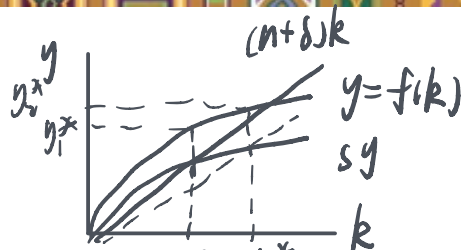
b) Total output grows to area $0D_2GL_2$. Returns to labor rise to represented by $0W_{M1}GL_2$, but note that the wage for each worker does not rise. Returns to capital increases to area $W_{M1}D_2G$.



c) Total output grows to area $0D_3HL_3$. Returns to labor rise to the area $0W_{M2}HL_3$. Notice that not only does total employment rise, the wage per worker also rises. Returns to capital increases to area $W_{M2}D_3H$. It is not clear whether returns to capitalists have increased, decreased, or have stayed the same (as compared with part b). In any case, returns to capitalists are now growing more slowly than before.



Exercise 3. Solow Growth Model



Specifically, graph output per worker, y , on the vertical axis, and capital stock per worker, k , on the horizontal. Draw in the production function $y = f(k)$, the investment function $s \cdot f(k)$, and the straight line that has slope equal to $(n + d)k$.

$$sy = (n + \delta)k \Rightarrow sf(k^*) = (\delta + n)k^*$$

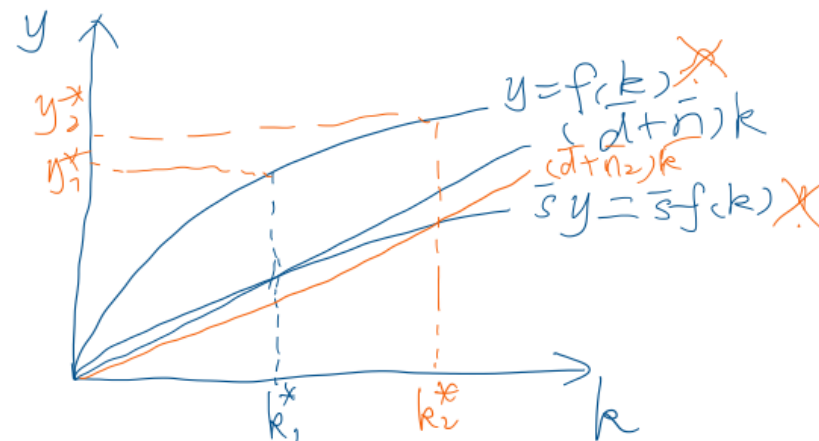
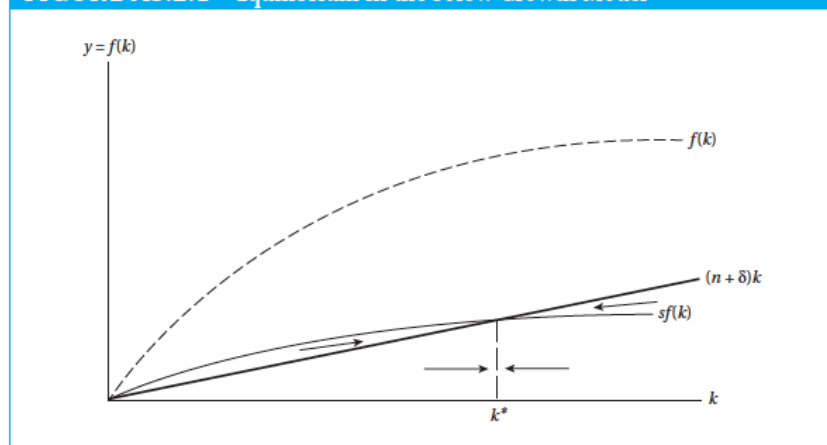
(a) What is a steady-state equilibrium? Mark the steady-state equilibrium on your graph as k_1^* .

(b) Suppose many years of hard work in population policy is finally paying off such that the population growth rate falls to n_2 . Mark the new steady-state equilibrium on your graph as k_2^* . Explain in words what has happened to this economy.

- $y_t = \frac{Y_t}{L_t} = \bar{A} \left(\frac{K_t}{L_t}\right)^\alpha \left(\frac{L_t}{L_t}\right)^{1-\alpha} = \bar{A}k^\alpha$
- Since $\Delta K_{t+1} = K_{t+1} - K_t = sY_t - dK_t$, and $k_t = \frac{K_t}{L_t}$
- Then $k_{t+1} - k_t = \frac{K_{t+1}}{L_{t+1}} - \frac{K_t}{L_t} = \frac{K_{t+1}}{(1+n)L_t} - \frac{K_t}{L_t} = \frac{K_{t+1} - K_t - nK_t}{(1+n)L_t} = \frac{sY_t - dK_t - nK_t}{(1+n)L_t} = \frac{sy_t - (d+n)k_t}{1+n}$
- a) In steady state, $sf(k^*) = (d+n)k^*$

b) The new steady state moves to the right. The economy goes to a higher level due to the population policy.

FIGURE A3.2.1 Equilibrium in the Solow Growth Model



1. Which of the following approaches does not offer an international dependence explanation of underdevelopment?

A. the false paradigm (虛假范例) model

B. the neoclassical counter-revolution

C. the dualistic (二元的) development model

D. the neocolonial dependence model

2. The supply curve of labor to industry in the Lewis model is horizontal if there is surplus labor in agriculture.

This condition persists as long as

A. the marginal product of labor is less than the average product of labor in agriculture.

B. the marginal product of labor in agriculture is less than the marginal product of labor in industry.

C. there are diminishing returns to labor in agriculture.

D. the marginal product of labor in agriculture is zero.